

ELECTRONIC SPECKLE METROLOGY

By

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by

Wilhelm Kurt Schwab

To my teacher,
William F. Carpenter

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ELECTRONIC SPECKLE METROLOGY

By

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Researchers at the University of South Carolina have developed Digital Correlation as a technique for the measurement of both two-dimensional and three-dimensional rigid and deformable body motion. Errors in displacement gradients as computed directly by Digital Correlation are generally on the order of 1000 micro strain. Penalty Method Finite Element smoothing has been used to generate displacement surfaces which may be analytically differentiated to compute displacement gradients more accurately. The primary disadvantage of the Digital Correlation method is that it is computationally intensive. Bruck developed a Newton-Raphson based iterative algorithm for computation of displacements and displacement gradients with fewer calculations than are required by the traditional coarse-fine search. The Newton-Raphson method still requires Finite Element smoothing to produce displacement gradients that are useable when actual displacement gradient component magnitudes are smaller than one percent.

The primary goal of the current study is to develop an iterative technique for the determination of in-plane displacements and strains with reduced computation. The Perturbation/Least-Squares (PLS) and Perturbation/Correlation (PC) algorithms are developed to allow the iterative computation of displacement components. Published results, and analytic models of the correlation process, developed in the current study, indicate that displacement gradients should generally not be computed by direct correlation. The program SPECKLE, developed by the author for the current study, computes strains from center-point displacements in regions sufficiently far apart that the strains can be computed accurately. Average normal strains are computed in three non-parallel directions and then transformed into normal strains and a shear strain using strain-transformation equations. The resulting technique avoids much of the computational overhead of Digital Correlation and does not require Finite Element smoothing.

The program SPECKLE is a Microsoft Windows program that runs on personal computers based on the Intel 80386 or higher processor. SPECKLE implements the PLS and PC methods of displacement computation, and normal and shear strain calculation based on strain transformation equations. Experiments performed in the current study indicate that SPECKLE may be used with an 8-bit frame grabber to measure normal and shear strains with maximum errors on the order of 100 micro strain.

INTRODUCTION

Experimental Stress Analysis is a broad discipline, a branch of which is concerned with the measurement of both stress and strain in structures and machines. The current study is concerned with the experimental determination of in-plane displacements and strains on the free surfaces of structural elements. Data is acquired and processed using a personal computer equipped with digital image capture hardware connected to a television system.

The Concepts Of Stress And Strain

Detailed descriptions of the theories of stress and strain are beyond the scope of this document; however, some general comments are appropriate. Stress is a measure of internal force per unit area in a continuous medium [Eisenberg, 1980, p. 57]. Strain is a purely geometric measure of deformation. Normal strains are related to the change of length per unit length, and shear strains are related to the reduction of what were originally right angles [Eisenberg, 1980, p. 78]. The in-plane strains on the free surface of a structural component are generally described by choosing a set of orthogonal coordinate axes in the plane and then indicating normal strains in each of the coordinate directions and a shear strain. These strains can be measured experimentally by measuring normal strains in three non-parallel directions in the plane. It is then possible to use transformation equations to determine the normal strains and the shear strain in any

desired coordinate system [Eisenberg, 1980, p. 158]. If the material in question is isotropic and its Poisson's ratio is known, then the normal strain in the direction perpendicular to the surface may be determined. The principal strains and maximum shear strain may then be computed [Dally and Riley, 1978, p. 320]. It must be mentioned that it is not possible to measure strain at a point. One must measure average normal strain over some length. As the length decreases the average strain approaches the value of the normal strain at the point of interest. Unfortunately, as the gage length decreases, so does the accuracy to which the average normal strain can be determined [Dally and Riley, 1978, p. 129].

Techniques Of Experimental Stress Analysis

This section provides a brief overview of some of the most common techniques for the measurement of in-plane strains for objects in states of plane stress.

Electrical Resistance Strain Gages

Electrical resistance strain gages are widely used to measure strains on free surfaces of structural elements. Strain gages can generally be mounted directly on a prototype component. It is also possible to incorporate strain gages into instruments such as load cells [Dally and Riley, 1978, p. 262]. The primary disadvantage of the use of strain gages is that one or more gages must be mounted at each point on the structural element under test at which the strain is to be determined. It is also necessary to know in advance the points at which strain must be measured.

Optical Methods

Many optical methods for the determination of both in-plane and out-of-plane displacements have been developed. Optical methods generally have the following advantages:

1. Optical methods generally involve the formation of an image of some type that allows data to be collected over a large region in one test.
2. Optical methods are non-contacting, so they do not alter the behavior of the test specimen.

Optical methods generally suffer the following disadvantages:

1. The information collected is in the form of interference fringes that might not be directly related to the value of the quantity of interest.
2. Optical methods often require the processing of a photographic film or plate before data can be extracted from the collected images.

The strengths and weaknesses of optical methods vary widely between methods and often must be assessed in terms of specific applications. An excellent example is provided by the fact that photoelasticity [Dally and Riley, 1978, p. 406] generally suffers the disadvantage that a model of the object under test must be constructed. This statement does not apply when the prototype material is a birefringent material, as is true for some transparent plastics.

Goal Of The Current Study

The introduction of computer hardware capable of capturing digital images from solid-state television cameras provides an opportunity for the development of new experimental techniques for the measurement of displacements and strains in solids. Interferometric optical methods of displacement measurement are sensitive to displacements on the order of the wavelength of light and often require vibration isolation. Many applications, however, do not require interferometric sensitivity. The goal of the current study is to develop a method for the determination of in-plane displacements and strains that does not require vibration isolation and exploits recent advances in personal computer technology. A secondary goal of the current study is to investigate the sensitivity of displacement gradients calculated via Digital Correlation methods [Chu et al., 1985] to changes in the computed rigid body displacement.

Previous Work

Electronic Speckle Metrology was first conceived as an electronic analog to Speckle Photography. As the work progressed it began to resemble the Digital Correlation and Automated Grid methods.

Speckle Photography And Computer Speckle Interferometry

Speckle Photography is a single-beam technique for the measurement of in-plane displacements. A single beam of coherent light is used to illuminate the object under test. The object is photographed in its undeformed configuration, loaded and photographed again on the same photographic plate. The resulting double exposure specklegram locally has the appearance of the sum of a speckle pattern with a translated copy of itself.

Interference fringes result when such a double exposure pattern is Fourier filtered [Burch and Tokarski, 1968]. Speckle Photography has been highly developed as an optical method for the measurement of displacements. Most applications of Speckle Photography do not require vibration isolation, which can be a distinct advantage over methods such as Holographic Interferometry [Jones and Wykes, 1989, p. 64]. Chiang et al. [1982] provide a detailed explanation of the application of Speckle Photography. Khetan and Chiang [1976] describe single aperture recording methods, and Chiang and Khetan [1979] describe multiple aperture recording methods for speckle photography. Strains are obtained by differentiation of the measured displacement fields [Khetan and Chiang, 1976]. Chen and Chiang [1990] developed an electronic analog of Speckle Photography, called Computer Speckle Interferometry. The book by Jones and Wykes [1989] provides an excellent description of optical methods for experimental stress analysis.

Digital Correlation Methods

Digital Correlation methods using white-light speckle patterns have been applied to both rigid body and deformable body mechanics problems, most prominently by researchers at the University of South Carolina. Peters and Ranson [1982] described the advantages of digital correlation over methods that require the formation and analysis of fringe patterns. Peters et al. [1982] used Digital Correlation to measure displacements, which were subsequently used as input to a boundary integral equation method to determine surface traction on a contour. Stresses within the boundary defined by the contour were computed from the surface tractions. Sirkis and Taylor [1990] used a

similar technique for Elastic-plastic stress analysis, and noted that bounds on the accuracy of displacement measurements made with digital computers should be expressed in terms of pixels for comparison. Peters et al. [1983] used Digital Correlation to solve problems in rigid body mechanics. Chu et al. [1985] applied Digital Correlation to deformable body mechanics, and noted that the computed displacement gradients were very sensitive to the computed values of the displacement components. Sutton et al. [1991] developed a method for smoothing the displacement fields obtained by correlation, allowing displacement gradients to be computed. The primary disadvantage of Digital Correlation methods is that they are computationally intensive. Bruck et al. [1989] developed a Newton-Raphson algorithm that reduces the amount of computation required to implement correlation methods. Chen and Chiang [1992] applied Fourier Transform analysis to the study of Displacement-only Digital Correlation with laser speckle patterns, arriving at theoretical limits for the accuracy of such correlation methods and bounds for allowable displacement gradient magnitudes.

Automated Grid Methods

Electronic Speckle Metrology is closely related to Automated Grid methods, which have been studied extensively. Sirkis [1990] provides a summary of previous work in grid methods. Sirkis [1990] also provides a detailed analysis of a particular Automated Grid method, and concludes that the fill factor, signal to noise ratio and grid spot diameter control the accuracy of the method considered. Sirkis and Taylor [1990] and Fail and Taylor [1990] developed experimental techniques, based on the use of CCD cameras,

which can be classified as grid methods. Sirkis and Lim [1991] studied the accuracy of automated grid methods.

Hybrid Methods

Optical methods often provide information in the form of fringe patterns, which must be analyzed to provide useful information. Many hybrid techniques that use image processing to analyze fringe patterns have been developed. Chen [1985] and Gillies [1988] applied image processing to the analysis of photoelastic fringes. Erbeck [1985] and Ineichen et al. [1980] applied image processing techniques to Speckle Photography fringes. Electronic Speckle Pattern Interferometry (ESPI) is a technique that combines an optical interferometer with a television system to eliminate the photographic processing step required by Speckle Photography. ESPI was first demonstrated by Butters and Leendertz, and is described in detail by Jones and Wykes [1989, p. 165].

Electronic Speckle Metrology

This section provides an overview of Electronic Speckle Metrology as implemented for the current study. The program SPECKLE, developed by the author to support the current study, implements the computational methods described in this document. The appendix contains a detailed description of SPECKLE's user interface.

Experimental Configuration

Electronic Speckle Metrology is based on the assumption that the test specimen has some intrinsic texture which moves with the test specimen during deformation. The intrinsic texture may be added as part of the specimen preparation, resulting in a

white-light speckle pattern [Asundi and Chiang, 1982], or it may be produced by laser speckle. The use of laser speckle proved to be impossible with the optics available to the author, so white-light speckle patterns were used in all experiments described in this document. It should be noted that laser speckle can be resolved by CCD cameras given appropriate optics [Chen and Chiang, 1992]. The experimental configuration includes the following hardware:

1. a light source
2. the test specimen and any required loading hardware
3. a television camera, frame grabber, and computer.

The camera should be oriented normal to the plane in which the test specimen is to be deformed. A schematic of the experimental configuration is shown in Figure 1.

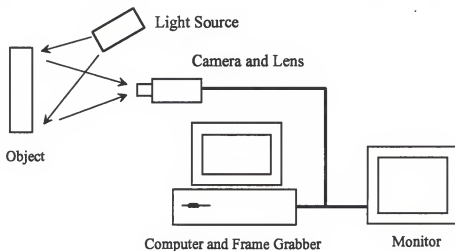


Figure 1. Experimental configuration.

The experimental procedure is essentially as follows:

1. Prepare the specimen for the test, and assemble the specimen loading apparatus.
2. Place any required pre-load on the test specimen.
3. Capture a digital image of the test specimen.
4. Load the test specimen as required to produce the desired deformation.
5. Capture a digital image of the test specimen in its deformed configuration.
6. Analyze the undeformed and deformed image pair using SPECKLE.

The SPECKLE Program

The program SPECKLE is a Microsoft Windows application, developed by the author for the current study, that provides a basic implementation of Electronic Speckle Metrology. Figure 2 shows the SPECKLE main window with its undeformed and deformed image viewers, which must always be open, and optional scan line and histogram windows. The user may size the windows freely, and use the scroll bars on the image viewers to view any desired portion of the image. SPECKLE uses the positions of the scroll bar thumbs to interpret mouse clicks by the user, so it is not necessary to scroll the two windows to the same location. The user may load undeformed and deformed images of an object and determine displacements and average strains in regions as desired. The user must indicate the locations of three features in both the undeformed and deformed images. The locations do not need to be precise since SPECKLE uses either the Perturbation/Least-Squares (PLS) or Perturbation/Correlation (PC) method, both of which are developed in this document, to improve the estimates of displacement resulting

from the user's input. The resulting displacements are used to determine the average strains in the region in which points were selected by the user

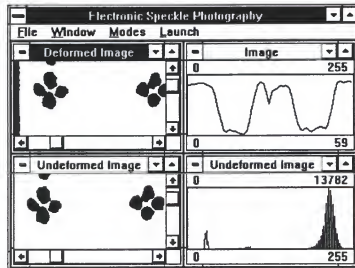


Figure 2. A view of the SPECKLE user interface.

Dynamic Histogramming

The images used in the current study were captured with the program DT2953, which was written by the author to support the Data Translation DT-2953 frame grabber for the IBM Micro Channel architecture. The program DT2953 allows the user to activate a real time histogram of a portion of the frame grabber. The histogram display is updated at approximately two hertz, allowing the user to set the light source intensity and camera aperture. The lower right window shown in Figure 2 contains a typical histogram for black dots on a white surface when the light level is chosen properly. Note that most of the range of the frame grabber is being used. Assuming that the test specimen is white with black dots, the light level is controlled by first aligning and focusing the camera and then adjusting the light source and lens aperture to admit as much light as possible without

clipping, which occurs when light intensity levels exceed the allowable input range. The adjustment is simple since the histogram responds to changes almost immediately. A typical dynamic histogram is shown in Figure 3. No peaks corresponding to dark features are visible because the scanned region does not include any such features. The most important regions to scan are the bright regions, so that clipping may be avoided.

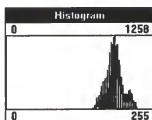


Figure 3. A dynamic histogram window.

Overview Of This Document

This section briefly describes the purpose of each of the chapters in the body of this document and of the appendix.

Basic Theory And Notation

The chapter "Basic Theory And Notation" describes a continuum model of the operation of the frame grabber and defines notation which is used in subsequent chapters. The fundamental assumption of the methods described in this document is that the image of the object deforms with the object. It is shown that this property is preserved when the frame grabber distorts the image.

Displacement Refinement

The chapter "Displacement Refinement" develops an algorithm for the iterative computation of displacements, and also develops the Perturbation/Least-Squares (PLS) and Perturbation/Correlation (PC) methods of displacement refinement.

Analytic Results On Errors

The chapter "Analytic Results On Errors" presents analytic studies of errors due to out-of-plane displacement and noise. It also explores the numerical conditioning of the use of correlation for the determination of displacement and displacement gradients. The models considered are all one-dimensional and idealized. The emphasis of the chapter is on obtaining qualitative results of the greatest possible generality; therefore, the numerical bounds obtained are often not precise.

Preparation For Experiments

The chapter "Preparation For Experiments" describes the calibration of the experimental system and the construction of test specimens and their loading or translating mechanisms.

Experimental Results

The chapter "Experimental Results" presents the results of experiments performed to verify the correctness of the PLS and PC methods, and the implementation of these methods in SPECKLE.

The SPECKLE User Interface

The Appendix provides a detailed description of SPECKLE from a user's perspective. It also describes the theory of SPECKLE's calibration method.

Summary

This chapter provides an overview of Experimental Stress Analysis, and the place of Electronic Speckle Metrology among other experimental methods. The current study has two distinct goals: the development of a new experimental method, and the study of sensitivities in existing methods. The review of literature presented in this chapter is extended throughout this document as appropriate.

BASIC THEORY AND NOTATION

The purpose of this chapter is to provide a foundation for the development of iterative displacement correction algorithms in subsequent chapters. Abstractions which facilitate the use of the experimental data are defined. In particular the concept of the motion function of Continuum Mechanics is extended to coordinates that are natural for use with the experimental data. The resulting continuum model is used as a guide for the interpretation of discrete experimental data throughout this document. The fundamental assumption of Electronic Speckle Metrology is that the image of the object deforms with the object. It is shown that this property is preserved when the idealized frame grabber distorts the images.

Mathematical Notation

This section provides a brief discussion of notation commonly used by mathematicians. The conventions described are used throughout this document to allow complicated expressions to be written in compact forms. The benefits of writing expressions using this notation is apparent in the chapters "Displacement Refinement" and "Analytic Results On Errors."

Function Definitions

We will consider a function to be a mapping from one set into another set [Bruck, 1978, p. 21]. Suppose that U and V are sets, and f is a function from U into V .

Then we will write $f : U \rightarrow V$. The element of V corresponding to a given element of U is sometimes indicated by the notation

$$\begin{aligned} f : U &\rightarrow V \\ u &\mapsto f(u) \end{aligned}$$

where $f(u)$ is replaced by the value given by f when applied to the element u . The set U is called the domain of f . The symbol $\mathfrak{R}$ is used to denote the set $(-\infty, \infty)$.

Sets And Images Of Sets

A common notation for sets is $\{ x : P(x) \}$, which is read as "the set of all x such that the property $P(x)$ is true" [Herstein, 1975, p. 2]. This notation allows very compact and readable statements. Another concept which is used in the developments to follow is that of the image of a set under a function. Let $f : U \rightarrow V$ be a given function, and let $W \subset U$. Then we define the image of W under f by

$$f(W) = \{ f(u) \in V : u \in W \}$$

which is the set of all values that result when f is applied to each element of U .

Operations On Functions

It is often convenient to consider functions which arise from combinations of existing functions. Let functions $f : F \rightarrow \mathfrak{R}$ and $g : G \rightarrow \mathfrak{R}$ be given. Define the sum of f and g by

$$\begin{aligned} f + g : F \cap G &\rightarrow \mathfrak{R} \\ x &\mapsto f(x) + g(x) \end{aligned}$$

The difference, product and quotient of real-valued functions are defined similarly.

Another very common way that new functions arise is by composition. Given the functions f and g defined above, define the composite function

$$\begin{aligned} f \circ g : H &\rightarrow \mathfrak{R} \\ x &\mapsto f(g(x)) \end{aligned}$$

where the set H is the set of all points x in the domain of G such that $g(x)$ is in the domain of f . The symbol $f \circ g$ may be read as "f compose g" or "f composed with g." Section 1.6 of the book by Leithold [1981] contains a description of the operations described in this section. Sum, difference, and composite functions and function inner products [Boyce and DiPrima, 1977, p. 463] are exploited throughout this document to simplify expressions involving integrals.

Topological Descriptions

In this section we formulate a continuum model of the operation of the frame grabber. This model will be applied, in subsequent chapters, to the measurement of the plane motion of a test specimen from images of the specimen in its undeformed and deformed configurations. We begin by considering the light intensity patterns to which the light sensitive elements of the camera are exposed and the mapping of these intensity patterns to gray levels in a logical image space on the frame grabber. We then describe conditions which allow us to compute motions using digital image coordinates. By analogy to the theory of continuum mechanics, we assume that all functions entering the development are continuous [Malvern, 1969, p. 1].

Spatial Relationships

The physical image plane is the plane containing the light sensitive surface of the camera. The logical image plane is the imaginary plane through the sampled data produced by the frame grabber. This section defines sets and functions which allow us to describe the spatial mapping of images from the physical image plane to the logical image plane. We begin with the following definitions:

1. The sets P and L are the physical and logical image planes, respectively.
2. The camera spatial function $c : P \rightarrow L$, assumed to be invertible and continuously differentiable, maps each point in the physical image plane to the corresponding point in the logical image plane.
3. $\hat{S}_A$ is the set of all points in the physical image plane such that the light intensity due to the test specimen in its undeformed configuration is non-zero.
4. $\hat{S}_B$ is the set of all points in the physical image plane such that the light intensity due to the test specimen in its deformed configuration is non-zero.
5. $S_A = c(\hat{S}_A)$ is the set of all points in the logical image of the test specimen in its undeformed configuration.
6. $S_B = c(\hat{S}_B)$ is the set of all points in the logical image of the test specimen in its deformed configuration.
7. We assume that there exists a unique invertible and continuously differentiable function $\hat{x} : \hat{S}_A \rightarrow \hat{S}_B$, called the physical image motion, that maps each point in the physical undeformed image to the corresponding point in the physical deformed image.
8. The logical image motion

$$\begin{aligned} x : S_A &\rightarrow S_B \\ X &\mapsto c \circ \hat{x} \circ c^{-1}(X) \end{aligned}$$

maps each point in the logical undeformed image to the corresponding point in the logical deformed image.

We make the additional assumptions that c^{-1} and $\hat{x}^{-1}$ are continuously differentiable.

Note that since c , c^{-1} and $\hat{x}$ are invertible we have

$$\begin{aligned} x^{-1} : S_B &\rightarrow S_A \\ X &:= c \circ \hat{x}^{-1} \circ c^{-1}(X) \end{aligned}$$

Since c , c^{-1} , $\hat{x}$, and $\hat{x}^{-1}$ are continuous and since compositions of continuous functions are continuous it follows that x and x^{-1} are continuous. The continuous differentiability of x and x^{-1} follows from the continuous differentiability of c , c^{-1} , $\hat{x}$, and $\hat{x}^{-1}$, and the chain rule. See Rudin [1964] for discussions of continuity, continuous differentiability and the chain rule. The sets and functions described above are shown in Figure 4.

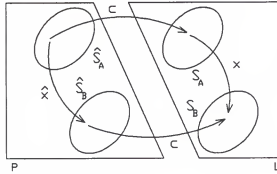


Figure 4. Spatial mapping from the physical to logical planes.

Description of Intensities

We now add descriptions of the luminous intensity patterns impinging on the light sensitive surface of the camera, and of the corresponding gray level patterns. The functions which must be added to the existing topological description are as follows:

1. The camera intensity function $\gamma : [0, \infty) \rightarrow [0, \infty)$, assumed to be invertible and continuously differentiable, maps luminous intensity values in the physical image plane to gray level values in the logical image plane.
2. The physical undeformed intensity $\hat{A} : \hat{S}_A \rightarrow [0, \infty)$, assumed to be continuously differentiable, describes the light intensity pattern impinging on the sensitive surface of the camera when the test specimen is in its undeformed configuration.
3. The physical deformed intensity $\hat{B} : \hat{S}_B \rightarrow [0, \infty)$, assumed to be continuously differentiable, describes the light intensity pattern impinging on the sensitive surface of the camera when the test specimen is in its deformed configuration.
4. The logical undeformed intensity

$$\begin{aligned} A : S_A &\rightarrow [0, \infty) \\ X &\mapsto \gamma \circ \hat{A} \circ c^{-1}(X) \end{aligned}$$

describes the gray level pattern that corresponds to the physical undeformed intensity.

5. The logical deformed intensity

$$\begin{aligned} B : S_B &\rightarrow [0, \infty) \\ X &\mapsto \gamma \circ \hat{B} \circ \hat{x} \circ c^{-1} \circ x^{-1}(X) \end{aligned}$$

describes the gray level pattern that corresponds to the physical deformed intensity.

Note that by the assumptions and results above and the chain rule, the functions A and B are continuously differentiable. The complete topological description is shown in Figure 5, which may be more easily understood by comparison with Figure 4.

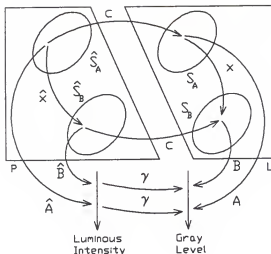


Figure 5. Topological description of intensities.

Note that the physical and logical images must satisfy the equations

$$\begin{aligned} A &= \gamma \circ \hat{A} \circ c^{-1} \\ B \circ x &= \gamma \circ \hat{B} \circ \hat{x} \circ c^{-1} \end{aligned}$$

and

$$\begin{aligned} \hat{A} &= \gamma^{-1} \circ A \circ c \\ \hat{B} \circ \hat{x} &= \gamma^{-1} \circ B \circ x \circ c \end{aligned}$$

The Use Of Logical Image Plane Coordinates

Experimental techniques which use photographs of a test specimen to measure its motion are based on the assumption that the image of the object deforms in a predictable way as the test specimen deforms. For the purposes of the current study we expect one of two scenarios:

1. If the light source intensity remains constant then we expect the physical deformed intensity composed with the physical image motion function to be equal to the physical undeformed intensity.
2. If the light source intensity changes between exposures then we expect the physical deformed intensity composed with the physical image motion function to be a scalar multiple to the physical undeformed intensity.

Both of the above statements assume that the object is plane, the object deforms in its plane, and that the physical image plane is parallel to the object plane. We now state and prove a Lemma specifying conditions which are sufficient to allow us to compute deformations in logical image coordinates.

Lemma. In terms of the notation described above

1. $B \circ x = A$ if and only if $\hat{B} \circ \hat{x} = \hat{A}$.
2. If the camera intensity function γ is linear then
 - i. If there exists a real number, a , such that $\hat{B} \circ \hat{x} = a\hat{A}$ then $B \circ x = aA$.
 - ii. If there exists a real number, a , such that $B \circ x = aA$ then $\hat{B} \circ \hat{x} = a\hat{A}$.

Proof. The proof of 1 is as follows:.

Suppose $\hat{B} \circ \hat{x} = \hat{A}$. Then the relations

$$A = \gamma \circ \hat{A} \circ c^{-1}$$

$$B \circ x = \gamma \circ \hat{B} \circ \hat{x} \circ c^{-1}$$

imply $B \circ x = A$. Suppose $B \circ x = A$. The the relations

$$\hat{A} = \gamma^{-1} \circ A \circ c$$

$$\hat{B} \circ \hat{x} = \gamma^{-1} \circ B \circ x \circ c$$

imply $\hat{B} \circ \hat{x} = \hat{A}$.

To prove 2.i, suppose there exists a real number, a , such that $\hat{B} \circ \hat{x} = a\hat{A}$. Note that the relations

$$\begin{aligned} A &= \gamma \circ \hat{A} \circ c^{-1} \\ B \circ x &= \gamma \circ \hat{B} \circ \hat{x} \circ c^{-1} \end{aligned}$$

and the linearity of γ imply

$$\begin{aligned} B \circ x &= \gamma \circ \hat{B} \circ \hat{x} \circ c^{-1} \\ &= \gamma \circ (a\hat{A}) \circ c^{-1} \\ &= a(\gamma \circ \hat{A} \circ c^{-1}) \\ &= aA \end{aligned}$$

The proof of 2.ii is similar. QED.

The above lemma states conditions sufficient to guarantee that the logical image deforms with the logical image motion if and only if the physical image deforms with the physical image motion. These results allow us to compute the motion of the object in the logical image plane without considering distortions caused by the camera and frame grabber. It is necessary to consider these distortions when interpreting the results in terms of the object.

Computational Descriptions

In this section we develop notation to simplify the use of input data in the form of digital images. Unless otherwise stated, all formulations are in terms of logical image coordinates.

The Point And Region Of Interest

The displacements and strains computed by the techniques described in this document are all computed as averages over a region. The region, called the Region of

Interest, is a square array of points and is specified by a center point, called the Point of Interest, and a linear dimension. We denote the Point of Interest by $X^{(0)}$ and the points in the Region of Interest are given by

$$X^{(m,n)} = X^{(0)} - \begin{pmatrix} m - \text{int}(R/2) \\ n - \text{int}(R/2) \end{pmatrix} \text{pixels} \quad \text{for all } m, n \in \{0, \dots, R-1\}$$

where R is the linear size of the Region of Interest, and int is the greatest integer function [Leithold, 1981, p. 44]. Note that the Point of Interest is truly the center of the region only if the region size is odd. For convenience we define the index set

$$\Lambda = \{(m,n) : m,n \in \{0, \dots, R-1\}\}$$

to index the Region of Interest. Note that $X^{(0)} \neq X^{(0,0)}$.

Sequence Of Approximate Motions

The techniques for the determination of displacement to be considered in this document are iterative. To facilitate the discussion of such schemes, we define a sequence of approximate motions of the form

$$\begin{aligned} x^{(k)} : S_A &\rightarrow S_B \\ X &\rightarrow X + u^{(k)} \end{aligned}$$

for all positive integers k , where the constant displacement vectors $u^{(1)}, u^{(2)}, \dots$ are to be determined.

Sampled Data

The governing equations for computation developed in the following chapter involve values of or integrals involving the functions A , and $B \circ x^{(k)}, D_1 B \circ x^{(k)}$, and $D_2 B \circ x^{(k)}$ for all positive integers, k . To facilitate the development of the governing

equations, define samples of the undeformed and deformed intensities and of the partial derivatives of the deformed intensity

$$\begin{aligned} A_{mn} &= A(X^{(m,n)}) \\ B_{mn}^{(k)} &= B \circ x^{(k)}(X^{(m,n)}) \\ D_1 B_{mn}^{(k)} &= D_1 B \circ x^{(k)}(X^{(m,n)}) \\ D_2 B_{mn}^{(k)} &= D_2 B \circ x^{(k)}(X^{(m,n)}) \end{aligned}$$

for all $(m, n) \in \Lambda$ and for all $k \in \{1, 2, \dots\}$. It is necessary to determine approximations to these values based on the experimental data. The remainder of this section defines quantities which may be used to approximate the samples of the logical intensities.

Digital Images

Consider the problem of approximating the logical intensities by data collected in the form of digital images. Let S be the set of all points at which sampled data is collected using the frame grabber. Let functions

$$\alpha, \beta : S \rightarrow [0, \infty)$$

describe the digital image intensities of the test specimen in its undeformed and deformed configurations, respectively. Let the functions

$$\tilde{A}, \tilde{B} : L \rightarrow \mathfrak{R}$$

be interpolated extensions of the functions α and β , respectively, where L is the logical image plane. The method of interpolation is arbitrary. SPECKLE currently supports only bilinear interpolation, but nonlinear interpolation methods are often preferable

[Sutton, 1988]. Define samples of the interpolated intensities by

$$\begin{aligned} \tilde{A}_{mn} &= \tilde{A}(X^{(m,n)}) \\ \tilde{B}_{mn}^{(k)} &= \tilde{B} \circ x^{(k)}(X^{(m,n)}) \end{aligned}$$

for all $(m, n) \in \Lambda$ and for all positive integers, k . For each positive integer, k , the approximations, $\Delta_1 \tilde{B}_{mn}^{(k)}$ and $\Delta_2 \tilde{B}_{mn}^{(k)}$ for all $(m, n) \in \Lambda$, to the partial derivatives of the deformed intensity are obtained either through finite difference approximations based on the values of $\tilde{B}_{mn}^{(k)}$ for all $(m, n) \in \Lambda$ or by differentiation of the interpolated functions.

Summary

This chapter provides the foundation for the iterative displacement computation algorithms to be developed in the next chapter. The collection of data using a television camera is described in terms of point set topology. The light intensity patterns on the light sensitive portion of the camera and their transformation into gray level patterns on a logical image plane coincident with the frame grabber are described using a continuum model. The chapter concludes with a development of notation to facilitate the use of digital images for computation.

DISPLACEMENT REFINEMENT

This chapter describes the Perturbation/Least-Squares (PLS) and Perturbation/Correlation (PC) methods for the determination of displacement in two dimensions. Both methods require a pair of images of the test specimen, one image each before and after deformation. The implementations of the two methods share many common features, and these common features have been used as a guide in the development of SPECKLE.

Iterative Computation Of Displacement

This section provides an overview of the iterative computation of displacement using digital images of a test specimen in its undeformed and deformed configurations. SPECKLE relies on the user to indicate a base displacement as an integral number of pixels. The user may use a mouse to click on a given feature in both the undeformed and deformed images or enter the displacement components in a dialog box. The details of the techniques for indicating a base displacement are described in detail in the Appendix. The estimated displacement provided by the user is refined through a sequence of iterations as follows:

1. Select the interpolation and differentiation algorithms, and the displacement refinement algorithm. SPECKLE currently supports only bilinear interpolation and a second order central difference differentiation algorithm. A displacement

refinement algorithm is selected by choosing the corresponding Displacement Engine. See the Appendix for details.

2. Initialize the approximate displacement $u^{(1)}$ using the estimate provided by the user. Use the undeformed and deformed intensity samples to initialize the values of $\tilde{A}_{mn}$ and $\tilde{B}_{mn}^{(1)}$ for all $(m, n) \in \Lambda$. Use the selected numerical differentiation algorithm to compute the values of $\Delta_1 \tilde{B}_{mn}^{(1)}$ and $\Delta_2 \tilde{B}_{mn}^{(1)}$ for all $(m, n) \in \Lambda$, based on the values of $\tilde{B}_{mn}^{(1)}$ for all $(m, n) \in \Lambda$. Set the iteration number, k , equal to one.
3. Compute $u^{(k+1)}$ using the selected displacement refinement algorithm. The displacement refinement algorithms available to users of SPECKLE are developed in this chapter.
4. If $|u^{(k+1)} - u^{(k)}|$ is sufficiently small or if the maximum allowed number of iterations has been exceeded then stop.
5. Use the interpolation algorithm to compute $\tilde{B}_{mn}^{(k+1)}$ for all $(m, n) \in \Lambda$, based on the values of $\tilde{B}_{mn}^{(1)}$ for all $(m, n) \in \Lambda$ and the value of $u^{(k+1)}$. Use the selected numerical differentiation algorithm to determine $\Delta_1 \tilde{B}_{mn}^{(k+1)}$ and $\Delta_2 \tilde{B}_{mn}^{(k+1)}$ for all $(m, n) \in \Lambda$, based on the values of $\tilde{B}_{mn}^{(k+1)}$ for all $(m, n) \in \Lambda$.
6. Increment k and go to Step 2.

The PLS and PC methods are described in terms of a single step that may be used in Step 2 of the displacement computation algorithm, a flow chart of which is shown in Figure 6.

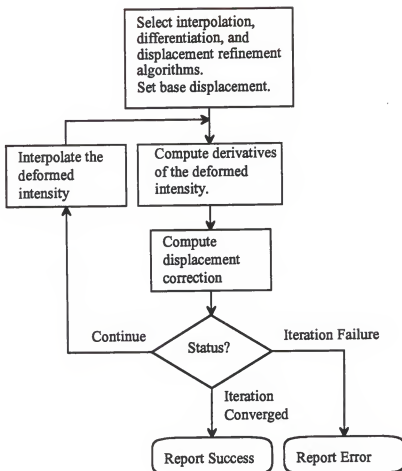


Figure 6. Flow chart of displacement iteration algorithm.

The Perturbation/Least-Squares Method

In this section we develop the PLS method governing equations, which are based on the assumption that the deformed intensity composed with the motion should equal the undeformed intensity. We use tangent plane approximations to the deformed intensity resulting in a perturbation equation, which relates the displacement components to the difference between the undeformed and deformed intensities. We then determine the required changes in the displacement components to minimize the squared error between the expected gray level differences and the observed gray level differences. The

development of the method begins with a heuristic argument and then proceeds to a precise development of a numerical algorithm. The notation developed in the chapter "Basic Theory and Notation" is used throughout the development.

Heuristic Development

Suppose that k steps of the displacement iteration have been completed, so that the displacement vectors

$$u^{(1)}, \dots, u^{(k)}$$

are known. Assume that the speckle pattern moves with the test specimen during deformation and that the light source output remains constant. Then we expect the undeformed and deformed intensities to satisfy

$$B \circ x = A$$

To improve the current estimate of displacement we seek the value of a constant displacement vector u such that the approximate equation

$$B(x^{(k)}(X) + u) \approx A(X)$$

is as true as possible throughout the Region of Interest. Substitution of a tangent plane approximation for the value of the deformed intensity results in the approximate equation

$$A(X) \approx B \circ x^{(k)}(X) + D_1 B \circ x^{(k)}(X)u_1 + D_2 B \circ x^{(k)}(X)u_2$$

for all X in the Region of Interest. The above motivates us to seek u such that

$$A_{mn} - B_{mn}^{(k)} \approx D_1 B_{mn}^{(k)}u_1 + D_2 B_{mn}^{(k)}u_2$$

in the Least Squares sense for all $(m, n) \in \Lambda$. To complete the iteration step, let

$$u^{(k+1)} = u^{(k)} + u$$

At this point control is returned to the displacement iteration algorithm.

Precise Formulation Of The PLS Method

Assume that k iterations of the displacement computation have been completed, so that the displacement vectors

$$u^{(1)}, \dots, u^{(k)}$$

are known. Using the preceding heuristic development as a guide, define matrices M , $M^{(1)}$ and $M^{(2)}$ by

$$M_{mn} = \tilde{A}_{mn} - \tilde{B}_{mn}^{(k)}$$

$$M_{mn}^{(1)} = \Delta_1 \tilde{B}_{mn}^{(k)}$$

$$M_{mn}^{(2)} = \Delta_2 \tilde{B}_{mn}^{(k)}$$

for all $(m, n) \in \Lambda$. Define a squared error function,

$$E : \mathfrak{R}^2 \rightarrow \mathfrak{R}$$

$$v : \rightarrow \left\| v_1 M^{(1)} + v_2 M^{(2)} - M \right\|^2$$

Let u be a displacement vector which satisfies

$$E(u) = \inf \{ E(v) : v \in \mathfrak{R}^2 \}$$

It can be shown that the space of n -dimensional matrices with real coefficients is an inner product space with inner product, $\cdot$, defined by

$$a \cdot b = \sum_{i=1}^n \left[\sum_{j=1}^n (a_{ij} b_{ij}) \right]$$

for all matrices a and b [Malvern, 1969, p 35] [Herstein, 1975, p. 191]. Hence it suffices

to solve the linear algebraic system [Davis, 1975, p. 176]

$$\begin{pmatrix} M^{(1)} : M^{(1)} & M^{(1)} : M^{(2)} \\ M^{(2)} : M^{(1)} & M^{(2)} : M^{(2)} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} M^{(1)} : M \\ M^{(2)} : M \end{pmatrix}$$

To complete the iteration step let

$$u^{(k+1)} = u^{(k)} + u$$

At this point control is returned to the displacement computation algorithm.

The Perturbation/Correlation Method

In this section we develop the Perturbation/Correlation (PC) method governing equations, which are based on the assumption that the deformed intensity composed with the motion is proportional to the undeformed intensity. An important advantage of Digital Correlation techniques is that they can be made tolerant of changes in light intensity between the capture of the undeformed and deformed images by normalization of the correlation function. The development of the method begins with a heuristic argument and then proceeds to a precise development of a numerical algorithm. The notation developed in the chapter "Basic Theory and Notation" is used throughout the development.

The Newton Raphson Method Applied To Digital Correlation

The primary disadvantage of Digital Correlation methods has been that they are computationally intensive. Bruck et al. [1989] developed a Newton-Raphson based algorithm for determining displacements and displacement gradients with reduced computation by digital correlation, but it requires the evaluation of first and second order derivatives of the correlation function. Displacement gradients must be on the order of 0.01 to be accurately determined by Bruck's method, unless a penalty method finite element smoothing scheme is used to smooth the computed displacements [Sutton, 1991]. The PC method also provides a differential correction algorithm for Digital Correlation, but requires estimates of only the first order partial derivatives of the sampled images. It should be noted that the current form of the PC method does not determine values of the displacement gradients.

Heuristic Development

We begin by applying the Schwarz Inequality to the undeformed intensity and a tangent plane approximation to the deformed intensity. The tangent plane approximation allows direct solution for a displacement correction which minimizes the difference between the two sides of the resulting inequality. For integrals of real-valued functions, a and b , the Schwarz Inequality has the form

$$\left(\int a(X)b(X)d\sigma \right)^2 \leq \left(\int a^2(X)d\sigma \right) \left(\int b^2(X)d\sigma \right)$$

with equality holding if and only if a and b are scalar multiples of one another. Let the functions A and B describe the undeformed and deformed intensities respectively, let $u^{(k)}$ be the approximate displacement vectors, and let the functions $x^{(k)}$ be the approximate motions. See the chapter "Basic Theory And Notation" for details. To develop the iterative algorithm, assume that k iterations have already been completed. Then the displacement vectors

$$u^{(1)}, \dots, u^{(k)}$$

are known. Let u be a constant displacement vector and note that

$$B(x^{(k)}(X) + u) \approx B \circ x^{(k)}(X) + D_1 B \circ x^{(k)}(X)u_1 + D_2 B \circ x^{(k)}(X)u_2$$

The Schwarz Inequality gives

$$\begin{aligned} & \left[\int A(X) \left(B \circ x^{(k)}(X) + D_1 B \circ x^{(k)}(X)u_1 + D_2 B \circ x^{(k)}(X)u_2 \right) d\sigma \right]^2 \\ & \leq \left[\int A^2(X)d\sigma \right] \left[\int \left(B \circ x^{(k)}(X) + D_1 B \circ x^{(k)}(X)u_1 + D_2 B \circ x^{(k)}(X)u_2 \right)^2 d\sigma \right] \end{aligned}$$

which, expressing the integrals as inner products, becomes

$$\begin{aligned} & \left[(A, B \circ x^{(k)}) + (A, D_1 B \circ x^{(k)})u_1 + (A, D_2 B \circ x^{(k)})u_2 \right]^2 \\ & \leq \|A\|^2 \|B \circ x^{(k)} + u_1 D_1 B \circ x^{(k)} + u_2 D_2 B \circ x^{(k)}\|^2 \end{aligned}$$

We seek the displacement, u , that minimizes the difference between the two sides of the

above inequality. Define an error function,

$$\begin{aligned} E : \mathfrak{R}^2 &\rightarrow [0, \infty) \\ v &: \rightarrow \|A\|^2 \|B \circ x^{(k)} + v_1 D_1 B \circ x^{(k)} + v_2 D_2 B \circ x^{(k)}\|^2 \\ &\quad - \left[(A, B \circ x^{(k)}) + (A, D_1 B \circ x^{(k)})v_1 + (A, D_2 B \circ x^{(k)})v_2 \right]^2 \end{aligned}$$

so that $E(u)$ represents the quantity to be minimized. The displacement vector u should be chosen such that

$$E(u) = \inf \{ E(v) : v \in \mathfrak{R}^2 \}$$

Hence we choose u such that

$$\begin{aligned} D_1 E(u) &= 0 \\ D_2 E(u) &= 0 \end{aligned}$$

It can be shown that under these conditions u must satisfy the following equations:

$$\begin{aligned} &\|A\|^2 (B \circ x^{(k)}, D_1 B \circ x^{(k)}) - (A, B \circ x^{(k)})(A, D_1 B \circ x^{(k)}) \\ &= \left[(A, D_1 B \circ x^{(k)})^2 - \|A\|^2 \|D_1 B \circ x^{(k)}\|^2 \right] u_1 \\ &\quad + \left[(A, D_2 B \circ x^{(k)})(A, D_1 B \circ x^{(k)}) - \|A\|^2 (D_2 B \circ x^{(k)}, D_1 B \circ x^{(k)}) \right] u_2 \end{aligned} \tag{1a}$$

$$\begin{aligned} &\|A\|^2 (B \circ x^{(k)}, D_2 B \circ x^{(k)}) - (A, B \circ x^{(k)})(A, D_2 B \circ x^{(k)}) \\ &= \left[(A, D_1 B \circ x^{(k)})(A, D_2 B \circ x^{(k)}) - \|A\|^2 (D_1 B \circ x^{(k)}, D_2 B \circ x^{(k)}) \right] u_1 \\ &\quad + \left[(A, D_2 B \circ x^{(k)})^2 - \|A\|^2 \|D_2 B \circ x^{(k)}\|^2 \right] u_2 \end{aligned} \tag{1b}$$

The above equations provide two simultaneous linear algebraic equations in the components of u . To complete the iteration step, determine the components of u which satisfy the above equations and let

$$u^{(k+1)} = u^{(k)} + u$$

At this point control is returned to the displacement iteration algorithm.

Precise Development Of The PC Method

Assume that k iterations of the displacement computation have been completed, so the displacement vectors

$$\mathbf{u}^{(1)}, \dots, \mathbf{u}^{(k)}$$

are known. Approximating the integrals in equations 1a and 1b by Riemann sums, let $\mathbf{u}$ be the displacement vector which satisfies the matrix equation $\mathbf{Mu} = \mathbf{C}$ where

$$\begin{aligned} \mathbf{C}_1 &= \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \right)^2 \right] \left[\Sigma \left(\tilde{\mathbf{B}}_{mn}^{(k)} \Delta_1 \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right] - \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right] \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \Delta_1 \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right] \\ \mathbf{C}_2 &= \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \right)^2 \right] \left[\Sigma \left(\tilde{\mathbf{B}}_{mn}^{(k)} \Delta_2 \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right] - \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right] \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \Delta_2 \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right] \\ \mathbf{M}_{11} &= \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \Delta_1 \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right]^2 - \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \right)^2 \right] \left[\Sigma \left(\Delta_1 \tilde{\mathbf{B}}_{mn}^{(k)} \right)^2 \right] \\ \mathbf{M}_{12} &= \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \Delta_1 \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right] \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \Delta_2 \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right] - \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \right)^2 \right] \left[\Sigma \left(\Delta_1 \tilde{\mathbf{B}}_{mn}^{(k)} \Delta_2 \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right] \\ \mathbf{M}_{21} &= \mathbf{M}_{12} \\ \mathbf{M}_{22} &= \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \Delta_2 \tilde{\mathbf{B}}_{mn}^{(k)} \right) \right]^2 - \left[\Sigma \left(\tilde{\mathbf{A}}_{mn} \right)^2 \right] \left[\Sigma \left(\Delta_2 \tilde{\mathbf{B}}_{mn}^{(k)} \right)^2 \right] \end{aligned}$$

and all sums are over all $(m, n) \in \Lambda$. To complete the iteration step, let

$$\mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + \mathbf{u}$$

At this point control is returned to the displacement computation algorithm.

Summary

This chapter describes the foundation of the displacement computation algorithm used by SPECKLE, beginning with a description of a high-level iterative algorithm for the computation of the average displacement in a specified region. The displacement computation algorithm is written in terms of an unspecified displacement refinement algorithm. The remainder of the chapter develops the PLS and PC methods for displacement refinement. Both methods use tangent plane approximations to allow direct

solution for an estimate of the required displacement correction. It is important to note that no conditions for convergence of the PLS and PC methods have been determined; however, the methods have performed well in practice, as results summarized in the chapter "Experimental Results" demonstrate. It is also important to note that both methods produce zero corrections if the exact motion is determined, as can be seen by substitution into the governing equations:

1. The PLS method produces a zero correction if $B \circ x^{(k)} = A$.
2. The PC method produces a zero correction if there exists a constant, a , such that $B \circ x^{(k)} = aA$.

ANALYTIC RESULTS ON ERRORS

This chapter represents an attempt to explain the errors which are inherent in the use of digital imaging systems to determine the motion of a test specimen from changes in observed gray levels. The chapter begins with a brief summary of published numerical and experimental work directed toward understanding errors in the use of vision systems in experimental solid mechanics. Qualitative analyses of the following are presented:

1. errors in normal strain due to out-of-plane displacement
2. errors in in-plane displacement measurements due to quantization errors and noise
3. sensitivity of the displacement gradients computed by the Digital Correlation method to changes in the assumed rigid body displacement.

One-dimensional continuum models are used throughout for simplicity. It should be emphasized that the purpose of all analyses presented in this chapter is to obtain general qualitative results. Combined numerical and experimental studies have been performed by Fail and Taylor [1990], Sirkis [1991], and Sutton et al. [1988]. A combined analytic and experimental study has been published by Chen and Chiang [1992].

Published Quantitative Results

The work published by Sirkis of current interest has been in the area of Automated Grid methods. Sirkis found that the fill factor, signal to noise ratio, and the diameter and

intensity profile of grid spots significantly affect the accuracy of Automated Grid methods [1990]. Additional numerical and experimental studies were performed by Fail and Taylor [1990]. Chen and Chiang [1992] used the Shannon Sampling Theorem to determine theoretical limits for the accuracy of displacements measured by Displacement-only Digital Correlation using laser speckle. Sutton et al. [1988] performed numerical and experimental analyses of errors in displacements and displacement gradients measured using the Digital Correlation method. Their conclusions included the following:

1. Up to a magnitude of 0.05, strains do not affect the accuracy of the computed center-point displacement when 12-bit intensity quantization is used.
2. To achieve optimal accuracy in both displacement and strain calculations, it is necessary to sample the signals at a frequency that is high relative to the frequency of the signal. It is also necessary to use at least a 12 bit A/D converter and to use nonlinear interpolation for sub-pixel intensity reconstruction.

The purpose of this chapter is to obtain qualitative results which are as independent of the exact form of the undeformed and deformed intensities as possible. Numerical estimates, when obtained, are not precise.

Effects Of Out-Of-Plane Displacement

It is clear that changes in apparent object size due to out-of-plane displacement should cause errors in normal strains measured using vision systems, unless the out-of-plane displacement is measured and considered as part of the analysis. The effects of out-of-plane displacement on the measurement of normal strains using vision systems

have been described by Sirkis and Lim [1991] and by Peters et al. [1989]. A stereoscopic vision system capable of measuring out-of-plane motions has been developed by Choa et al. [1989]. This section presents an order of magnitude analysis of the effects of out-of-plane displacement on measured normal strains when the out-of-plane displacement is not considered in the computation. The chapter "Experimental Results" describes an experiment which verifies the analysis of this section.

Measurement Of Strain Using A Pinhole Camera

Consider a pinhole camera imaging an object as shown in Figure 7.

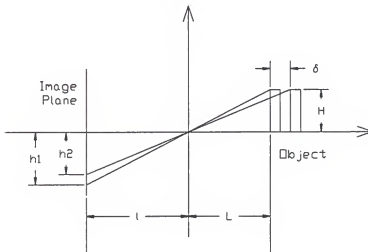


Figure 7. Definition sketch for out-of-plane displacement effects.

The object is moved a distance δ away from the pinhole between exposures. The heights of the image of the object before and after deformation are h_1 and h_2 respectively. Let $\Delta h = h_2 - h_1$, let $h = h_1$, and note that by similar triangles we have

$$\frac{H}{L} = \frac{h}{l}$$

and

$$\frac{H}{L+\delta} = \frac{h+\Delta h}{l}$$

The measured strain, ϵ , is then given by

$$\begin{aligned}\epsilon &= \frac{\Delta h}{h} \\ &= \frac{-\delta}{L+\delta} \\ &\approx -\frac{\delta}{L}\end{aligned}\tag{2}$$

which provides an estimate of the error to be expected in normal strain components, since the body did not deform.

Pinhole Equivalents Of Optical Systems

We now consider a method for approximating optical systems by pinholes to simplify estimates of the errors, due to out-of-plane displacements, in normal strains as measured using vision systems. Consider a lens being used to form an image of an object as shown in Figure 8. Suppose that the field of view has width W and the image has width w . Note that $D=L+l$ and that by similar triangles we have

$$\frac{W}{L} = \frac{w}{l}$$

Hence the distance, L , from the object to the pinhole which produces an image of the same size as the original lens is given by

$$L = \frac{D}{1 + \frac{w}{W}}\tag{3}$$

where D is the distance from the object plane to the image plane.

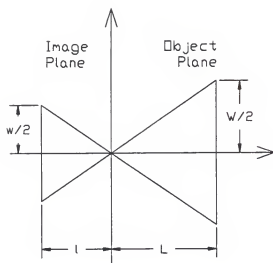


Figure 8. Definition sketch for the pinhole equivalent to a lens.

Order Of Magnitude Analysis Of Displacement Accuracy

The purpose of this section is to provide an order of magnitude analysis of the errors in displacement measurement due to noise. We begin by considering the effect of noise in intensity data on the accuracy with which we may determine displacements. Consider a one-dimensional image with a linear gray level distribution as shown in Figure 9.

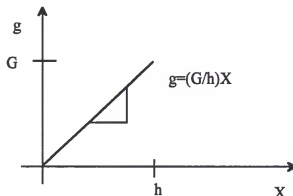


Figure 9. Change In Gray Level And Apparent Change In Position.

Note that a change in gray level Δg corresponds to a change in horizontal position ΔX , and

$$\frac{\Delta X}{h} = \frac{\Delta g}{G}$$

by similar triangles. We expect the error in displacement to be bounded as follows:

$$|\Delta u| \leq 2 |\Delta X|$$

$$\leq 2 \frac{h}{G} |\Delta g|$$

(4)

Using typical scales from current research, we choose $\Delta g = 1 \text{ gl}$, $h = 10 \text{ pixels}$, and

$G = 100 \text{ gl}$. We then have

$$|\Delta u| \leq 2 \frac{10 \text{ pixels}}{100 \text{ gl}} (1 \text{ gl})$$

$$\leq 0.2 \text{ pixels}$$

The next section of this chapter helps to explain how we are able to obtain displacements more accurately than this conservative estimate might lead us to expect.

Dependence Of Displacement On Noise Magnitude

In this section we consider an idealized one-dimensional example involving a rigid body displacement with noise superimposed on the image intensities. We will investigate the dependence of the displacement determined by correlation on the amplitude of the noise. The measured displacement is assumed to depend analytically on the noise amplitude, and all functions are assumed to be continuously differentiable. We will use implicit differentiation to compute the derivative of the measured displacement with respect to a measure of the noise amplitude, and then perform an asymptotic analysis of the resulting derivative for the case of sinusoidal noise functions of high frequency.

One-Dimensional Images And Motions

Consider a one-dimensional body which occupies positions described by the sets $S_A = [X_0 - h, X_0 + h]$ and S_B in its undeformed and deformed configurations, respectively. Let the function $B : S_B \rightarrow \mathfrak{R}$ describe the deformed intensity. For simplicity, we will consider a rigid motion. Fix a displacement, $\hat{u}$, and let the motion be

$$\begin{aligned}x &: S_A \rightarrow S_B \\ X &: \rightarrow X + \hat{u}\end{aligned}$$

Let the undeformed intensity be

$$\begin{aligned}A &: S_A \rightarrow \mathfrak{R} \\ X &: \rightarrow B \circ x(X) + \alpha n(X)\end{aligned}$$

where α , called the noise parameter, is a measure of the noise amplitude and $n : S_A \rightarrow \mathfrak{R}$ is a zero-mean noise shape function. The undeformed intensity is thus equal to the deformed intensity composed with the motion when the noise amplitude is zero. For each displacement, u , define the approximate motion

$$\begin{aligned}x_u &: S_A \rightarrow \mathfrak{R} \\ X &: \rightarrow X + u\end{aligned}$$

Comments On Scales

The functions defined above describe the undeformed and deformed image intensities and a noise superimposed on the undeformed image. The actual noise superimposed on the undeformed intensity is given by αn . Later in this section we will express the displacement determined by correlation as a power series in the parameter α , and neglect terms of second and higher order in α . We will assume that the values of A , B , and $|n|$ vary from zero to two hundred. A noise of two gray levels is thus obtained by choosing α on the order of 0.01, which should be sufficiently small to justify neglecting

the higher order terms. The size scales have been chosen to match typical values from the experiments performed as part of the current study.

Correlation Function And Measured Displacement

Define an inner product and a norm by

$$(a, b) = \int_{S_A} a(X)b(X)dX$$

$$\|a\| = \sqrt{(a, a)}$$

for all integrable $a, b : S_A \rightarrow \mathfrak{R}$. For each $\alpha \in \mathfrak{R}$ define a correlation function

$$C_\alpha : \mathfrak{R} \rightarrow \mathfrak{R}$$

$$u : \rightarrow \frac{(A, B \circ x_u)^2}{\|A\|^2 \|B \circ x_u\|^2}$$

The measured displacement is found by choosing the displacement which maximizes the above correlation function. Assume there exists a unique function $U : \mathfrak{R} \rightarrow \mathfrak{R}$ such that

$$C_\alpha(U(\alpha)) = \max C_\alpha$$

for all $\alpha \in \mathfrak{R}$. Given a noise parameter, α , $U(\alpha)$ is the measured displacement. Note that in particular $x_{U(0)} = x$, which states that the measured displacement is equal to the exact displacement when the noise amplitude is zero.

Determination Of The Displacement Estimate

The goal of this section is to determine how the measured displacement varies for small values of the noise parameter, α . The measured displacement may be written as

$$U(\alpha) = \hat{u} + U'(0)\alpha + O(\alpha^2)$$

(5)

Recall that the scales have been chosen so that a two gray level noise results when $\alpha=0.01$. The equation which implicitly defines the estimated displacement is

$$C'_\alpha(U(\alpha)) = 0$$

To facilitate the computation of the derivative of the correlation function, define for each $\alpha \in \mathfrak{R}$ functions

$$\begin{aligned} f_\alpha : \mathfrak{R} &\rightarrow \mathfrak{R} \\ u &\mapsto (B \circ x + \alpha n, B \circ x_u)^2 \\ g_\alpha : \mathfrak{R} &\rightarrow \mathfrak{R} \\ u &\mapsto \|B \circ x + \alpha n\|^2 \|B \circ x_u\|^2 \end{aligned}$$

so that $C_\alpha = \frac{f_\alpha}{g_\alpha}$. It can be shown that

$$\begin{aligned} f'_\alpha(u) &= 2(B \circ x + \alpha n, B \circ x_u)(B \circ x + \alpha n, B' \circ x_u) \\ g'_\alpha(u) &= 2\|B \circ x + \alpha n\|^2 (B \circ x_u, B' \circ x_u) \end{aligned}$$

The quotient rule for differentiation gives

$$C'_\alpha(u) = \frac{f'_\alpha(u)g_\alpha(u) - f_\alpha(u)g'_\alpha(u)}{g_\alpha^2(u)}$$

Since C_α has an extreme value at $U(\alpha)$, we have $C'_\alpha(U(\alpha)) = 0$, which implies

$$f'_\alpha(U(\alpha))g_\alpha(U(\alpha)) = f_\alpha(U(\alpha))g'_\alpha(U(\alpha))$$

Substitution of the expressions for f'_α and g'_α gives

$$(B \circ x + \alpha n, B' \circ x_{U(\alpha)})\|B \circ x_{U(\alpha)}\|^2 = (B \circ x + \alpha n, B \circ x_{U(\alpha)})(B \circ x_{U(\alpha)}, B' \circ x_{U(\alpha)})$$

which implicitly defines the measured displacement in terms of the noise parameter.

Differentiation with respect to α and subsequent evaluation when $\alpha=0$ results in

$$\begin{aligned} &[(n, B' \circ x) + (B \circ x, B'' \circ x)U'(0)]\|B \circ x\|^2 \\ &+ (B \circ x, B' \circ x)[2(B \circ x, B' \circ x)U'(0)] \\ &= [(n, B \circ x) + (B \circ x, B' \circ x)U'(0)](B \circ x, B' \circ x) \\ &+ \|B \circ x\|^2[(B' \circ x)^2 + (B \circ x, B'' \circ x)]U'(0) \end{aligned}$$

which yields

$$U'(0) = \frac{\|B \circ x\|^2 (n, B' \circ x) - (B \circ x, B' \circ x)(n, B \circ x)}{\|B \circ x\|^2 \|B' \circ x\|^2 - (B \circ x, B' \circ x)^2} \quad (10)$$

Note that the Schwarz Inequality gives

$$\|B \circ x\|^2 \|B' \circ x\|^2 \geq (B \circ x, B' \circ x)^2$$

with equality holding if and only if $B \circ x$ and $B' \circ x$ are proportional. Hence $U'(0)$ is defined provided B and B' are not proportional. If B and B' are proportional then $U'(0)$ is of the form $0/0$, but should still be definable as a limit.

Interpretation Of The Derivative

We now consider the effects of sinusoidal noise functions of large frequency, through an asymptotic analysis of $U'(0)$. The resulting expression is then used to provide an order of magnitude estimate of the displacement error observed in the use of vision systems for the measurement of plane motion. It will be seen that the effects of noise on the measured displacement decrease as the frequency of the noise increases.

Asymptotic Analysis. Let $B_0=200$ represent a scale for the image intensities. The noise function is then assumed to be of the form

$$\begin{aligned} n : S_A &\rightarrow \mathfrak{R} \\ X &\rightarrow B_0 \sin(\omega X) \end{aligned}$$

where ω is a constant spatial frequency. By successive integration by parts, it can be shown that for all sufficiently smooth functions f

$$(n, f) = O\left(\frac{1}{\omega}\right)$$

as $\omega \rightarrow \infty$ [Bleistein and Handelsman, 1986, p. 80]. Hence we have

$$U'(0) = O\left(\frac{1}{\omega}\right) \quad \text{as } \omega \rightarrow \infty$$

This result is consistent with the findings, in a particular example, of Sutton et. al. [1988] that rapid sampling is essential for accurate results.

Leading Terms. Inspection of the asymptotic expansion of the integral of the product of a sinusoid of large frequency and a smooth function [Bleistein and Handelsman, 1986, p. 80] leads to the order of magnitude estimates

$$\begin{aligned} |(n, B \circ x)| &\sim B_0^2 \frac{1}{\omega} \\ |(n, B' \circ x)| &\sim B_0 B'_0 \frac{1}{\omega} \end{aligned}$$

for large ω , where B'_0 is a characteristic value for the derivative of B , which is assumed to satisfy $B_0 \sim B'_0 h$. The magnitudes of the remaining integrals appearing in Equation 10 may be estimated as follows:

$$\begin{aligned} \|B \circ x\|^2 &\sim B_0^2 h \\ \|B' \circ x\|^2 &\sim [B'_0]^2 h \\ |(B \circ x, B' \circ x)| &\sim B_0 B'_0 h \end{aligned}$$

The above estimates indicate that

$$|U'(0)| \sim \frac{B_0}{B'_0 h \omega}$$

which gives

$$|U'(0)| \sim \frac{1}{\omega}$$

The frequency of the noise may be more conveniently expressed in terms of the number of cycles in the interval of interest, which we denote by N . The frequency then becomes

$\omega = \frac{\pi N}{h}$ and hence, for large values of N ,

$$|U'(0)| \sim \frac{h}{\pi N}$$

Numerical estimate of error. The author's experience indicates that the difference in gray level between two successive images of an object in the same configuration is typically two gray levels. The corresponding value of the noise parameter is $\alpha=0.01$. Since the effects of noise decrease with increasing frequency of noise, any estimates of the effect of noise should be based on a low frequency noise. Assume that the noise oscillates only twice in the correlation interval, so $N=2$, and assume the half gage length is given by $h=10$ pixels. The estimate of the error in displacement, $|\Delta u|$ is then given by

$$\begin{aligned} |\Delta u| &= \alpha |U'(0)| \\ &\sim 0.01 \frac{h}{\pi N} \\ &\sim 0.02 \text{ pixels} \end{aligned}$$

The use of the large frequency expansion for $|U'(0)|$ requires justification.

Figure 11 shows a graph, created by Mathcad, which compares the exact value of $|U'(0)|$ and the asymptotic estimate, for large N , given by $|U'(0)| \sim \frac{h}{\pi N}$ versus the number of noise oscillations, N . The example was computed assuming a parabolic image and a sinusoidal noise function. The comparison is favorable even at one oscillation over the interval.

The results of this section indicate that displacement errors on the order of 0.02 pixels are expected given the region size typically used for Digital Correlation, and the noise characteristics of the CCD camera and 8-bit frame grabber used to collect data for the current study.

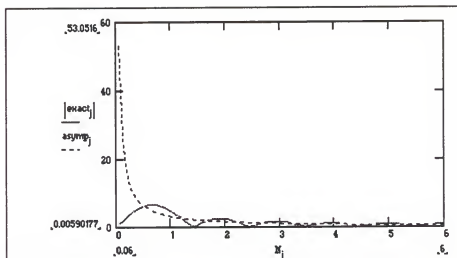


Figure 11. A test of the asymptotic estimate of Equation 6.

Uncertainty Relation For Correlation

This section represents an attempt to explain the strong dependence of strain components determined by Digital Correlation on the estimate of displacement, as observed by Chu et al. [1985] and by Sutton et al. [1991]. We will fix undeformed and deformed images in one dimension, and define a class of approximate motions functions, parameterized in terms of the approximate displacement and strain. We will then define a correlation function and study the relationship between changes in the approximate strain and displacement near maxima of the correlation function. The result will be an inequality relating the uncertainties in the approximate displacement and strain, and the size of the region over which the correlation is performed. Note that in the following analysis we will assume that all functions considered are continuously differentiable.

One Dimensional Images And Motions

Choose a position X_0 and a length h . Consider a one-dimensional body which occupies positions described by the sets $S_A = [X_0 - h, X_0 + h]$ and S_B in its undeformed and deformed configurations, respectively. For all displacements, u , and for all strains, ϵ , define an approximate motion

$$\begin{aligned} x_\epsilon^u &: S_A \rightarrow \mathfrak{R} \\ X &\mapsto X + u + \epsilon[X - X_0] \end{aligned}$$

Let the functions

$$\begin{aligned} A &: S_A \rightarrow \mathfrak{R} \\ B &: S_B \rightarrow \mathfrak{R} \end{aligned}$$

be the undeformed and deformed intensities, respectively.

The Correlation Function And Inner Products

Define a correlation function

$$\begin{aligned} C &: \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R} \\ (u, \epsilon) &\mapsto \frac{\left(\int_{S_A} A(X) B(x_\epsilon^u(X)) dX \right)^2}{\left(\int_{S_A} A^2(X) dX \right) \left(\int_{S_A} B^2(x_\epsilon^u(X)) dX \right)} \end{aligned}$$

which must be maximized to determine the displacement and strain. The analysis to follow is facilitated by the use of inner product notation. Let a and b be functions defined on the set S_A . Define inner products

$$\begin{aligned} (a, b) &= \int_{S_A} a(X) b(X) dX \\ \langle a, b \rangle &= \int_{S_A} a(X) b(X) (X - X_0) dX \end{aligned}$$

and a norm

$$\|a\| = \sqrt{(a, a)}$$

for all integrable $a, b : S_A \rightarrow \mathfrak{R}$. Using the above notation the correlation function may be

written as

$$C(u, \varepsilon) = \frac{(A, B \circ x_\varepsilon^u)^2}{\|A\|^2 \|B \circ x_\varepsilon^u\|^2} \quad \text{for all } (u, \varepsilon) \in \mathfrak{R} \times \mathfrak{R}$$

We now state and prove an important property of the inner products we have defined.

Lemma. Let $a, b : S_A \rightarrow \mathfrak{R}$ be integrable. Then

$$|\langle a, b \rangle| \leq h|(a, b)|$$

Proof. Note that since

$$\begin{aligned} S_A &= \{X \in \mathfrak{R} : X_0 - h \leq X \leq X_0 + h\} \\ &= \{X \in \mathfrak{R} : |X - X_0| \leq h\} \end{aligned}$$

we have

$$-h \int a(X)b(X)dX \leq \int a(X)b(X)(X - X_0)dX \leq h \int a(X)b(X)dX$$

where all integrals are over the set S_A . Then

$$-h(a, b) \leq \langle a, b \rangle \leq h(a, b)$$

which is equivalent to the result to be proven.

Differentiation Of The Correlation Function

The analysis to follow requires the first partial derivatives of the correlation function with respect to both strain and displacement. To facilitate the calculation of these derivatives, define functions

$$\begin{aligned} f : \mathfrak{R} \times \mathfrak{R} &\rightarrow \mathfrak{R} \\ (u, \varepsilon) &\rightarrow (A, B \circ x_\varepsilon^u)^2 \\ g : \mathfrak{R} \times \mathfrak{R} &\rightarrow \mathfrak{R} \\ (u, \varepsilon) &\rightarrow \|A\|^2 \|B \circ x_\varepsilon^u\|^2 \end{aligned}$$

so that $C=f/g$. It can be shown that

$$\begin{aligned} D_1 f(u, \varepsilon) &= 2(A, B \circ x_\varepsilon^u)(A, B' \circ x_\varepsilon^u) \\ D_2 f(u, \varepsilon) &= 2(A, B \circ x_\varepsilon^u)(A, B' \circ x_\varepsilon^u) \\ D_1 g(u, \varepsilon) &= 2\|A\|^2 (B \circ x_\varepsilon^u, B' \circ x_\varepsilon^u) \end{aligned}$$

$$D_2 g(u, \varepsilon) = 2 \|A\|^2 \langle B \circ x_g^u, B' \circ x_g^u \rangle$$

Sensitivity Analysis

In the following analysis we assume that the solution for the maximum of the correlation is not exact. The expressions below are not defined if the solution is exact. The implications of the indeterminacy at the exact solution of the estimates obtained are considered below. Suppose that numerical maximization of the correlation function value has resulted in the approximate displacement u and the approximate strain ε . To estimate the sensitivity of the strain computed by the correlation algorithm to the assumed rigid body displacement, u , change the displacement by an amount Δu and compute the resulting change, $\Delta \varepsilon$, in the computed strain assuming that the value of the correlation function remains constant at first order. The requirement that the value of the correlation function be constant at first order implies

$$0 = D_1 C(u, \varepsilon) \Delta u + D_2 C(u, \varepsilon) \Delta \varepsilon$$

which implies that

$$\begin{aligned} \left| \frac{\Delta \varepsilon}{\Delta u} \right| &= \left| \frac{D_1 C(u, \varepsilon)}{D_2 C(u, \varepsilon)} \right| \\ &= \left| \frac{D_1 f(u, \varepsilon) g(u, \varepsilon) - f(u, \varepsilon) D_1 g(u, \varepsilon)}{D_2 f(u, \varepsilon) g(u, \varepsilon) - f(u, \varepsilon) D_2 g(u, \varepsilon)} \right| \end{aligned}$$

Substitution of the expressions for the partial derivatives gives

$$\left| \frac{\Delta \varepsilon}{\Delta u} \right| = \frac{(A, B' \circ x_g^u) \|B \circ x_g^u\|^2 - (A, B \circ x_g^u) (B \circ x_g^u, B' \circ x_g^u)}{\langle A, B' \circ x_g^u \rangle \|B \circ x_g^u\|^2 - (A, B \circ x_g^u) \langle B \circ x_g^u, B' \circ x_g^u \rangle}$$

which may be written as

$$\begin{aligned} \left| \frac{\Delta \varepsilon}{\Delta u} \right| &= \left| \frac{(\|B \circ x_g^u\|^2 A, B' \circ x_g^u) - \langle (A, B \circ x_g^u) B \circ x_g^u, B' \circ x_g^u \rangle}{\langle \|B \circ x_g^u\|^2 A, B' \circ x_g^u \rangle - \langle (A, B \circ x_g^u) B \circ x_g^u, B' \circ x_g^u \rangle} \right| \\ &= \left| \frac{(\|B \circ x_g^u\|^2 A - (A, B \circ x_g^u) B \circ x_g^u, B' \circ x_g^u)}{\langle \|B \circ x_g^u\|^2 A - (A, B \circ x_g^u) B \circ x_g^u, B' \circ x_g^u \rangle} \right| \end{aligned}$$

Thus by the above Lemma we have the strain-displacement uncertainty relation

$$|\Delta \varepsilon| \geq \frac{|\Delta u|}{h} \quad (7)$$

The implications of this result are discussed below.

Implications for the computation of strain. Typical values from current research indicate that the uncertainty in displacement should be approximately 0.01 pixels and the gage length should be approximately twenty pixels. Under these conditions the strain-displacement uncertainty relation predicts an uncertainty in strain on the order of 1000 micro strain. This agrees well with the standard deviation of 0.007 found by Bruck et al. [1989]. It should be noted that the strain-displacement uncertainty relation provides a lower bound for the uncertainty in strain. It does not indicate that it is impossible to determine strain accurately, but it does indicate that one's confidence in the strain estimates obtained should be low.

Implications for the computation of displacement. The strain-displacement uncertainty relation can be written as

$$|\Delta u| \leq h |\Delta \varepsilon|$$

which provides an upper bound for the uncertainty in displacement given an uncertainty in strain. Note that a strain uncertainty $|\Delta \varepsilon|=0.001$ results in an estimate for the uncertainty in displacement of the form

$$|\Delta u| \leq 0.01 \text{ pixels}$$

given a gage length of twenty pixels, which corresponds to $h=10$ pixels. Hence, when computing displacements, it may be advisable to simply ignore the effects of strain at or

below the 1000 micro strain level, since the errors induced should be smaller than those resulting from noise and computational errors. Once the displacements have been determined to a known accuracy, it is possible to select sufficiently long gage lengths to determine average strains to a prescribed tolerance based on center point displacements.

Comments On The Indeterminacy At The Exact Solution

The strain-displacement uncertainty relation is not defined if the motion determined by the correlation algorithm maximizes the correlation function. The ratio of strain error to displacement error is of the form $0/0$ when the correlation function is maximized. This is expected, since we have asked how we must change the strain in response to changes in displacement in order to maintain a constant value of the correlation function. When the solution for the displacement and strain which maximize the value of the correlation function is exact, it should not be possible to alter the strain in response to a change in displacement in such a way that the correlation function value is held constant.

Summary

This chapter contains qualitative analyses of the effects of out-of-plane displacement on normal strains and the effects of noise on the accuracy with which displacements may be determined using correlation. An analysis of the simultaneous determination of displacement and strain by correlation is also presented. The results obtained indicate the following:

1. The error of measured displacements due to noise, neglecting cancellation, can easily approach 0.1 pixels when eight bit quantization is used.
2. The effect of noise on the displacement computed by correlation decreases as the noise frequency increases.
3. Displacement errors on the order of 0.02 pixels are expected given typical conditions for current applications of Digital Correlation.
4. Given the fact that Digital Correlation has a displacement uncertainty of 0.01 pixels, the expected strain uncertainty is on the order of 1000 micro strain when correlation is performed over a region with a radius of approximately ten pixels.

It is important to note that the analyses presented in this chapter do not consider the effects of interpolation. The lack of consideration of interpolation is important for the following reasons:

1. An important source of error has not been included in the models.
2. The error estimates presented in this chapter represent "best case" results, and it should not be possible to reduce errors below the predicted levels by improved interpolation algorithms.

PREPARATION FOR EXPERIMENTS

This chapter describes the preparation for experiments performed to provide verification of the PLS and PC methods of displacement refinement. The discussion begins with a description of the procedure used to calibrate SPECKLE's aspect ratio correction mechanism. The construction of the test specimens and their translation or loading mechanisms is then described.

Experimental Configuration

This section describes the basic experimental configuration used for all experiments described in this document. The television system is described in detail and then the basic experimental system is described.

Computer And Television System Details

The computer system consisted of an IBM PS/2 Model 70 B21 (Intel 80486 CPU) personal computer with 4MB RAM and the following components:

1. one 120MB hard drive
2. two 3.5 inch internal high density floppy disk drives and one external 5.25 inch high density floppy disk drive
3. one IBM 8514/A video adapter and one IBM 8514 monitor

4. one Data Translation DT-2953 frame grabber for the IBM Microchannel Architecture
5. one Sony XC-77CE CCD television camera with PS-12SU power supply
6. one Sony PVM-1342Q video monitor
7. one IBM 4019 Laser Printer and a Microsoft serial mouse.

The video system connections are shown in detail in Figure 12.

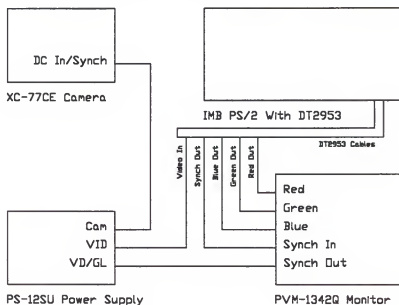


Figure 12. Video system connections.

Experimental Configuration And Procedures

The experimental technique described in this document is similar to both the Digital Correlation [Sutton, 1991] and Automated Grid methods [Sirkis, 1991]. The differences are in the method of computation and the preparation of the test specimen surface.

Experimental configuration. The object under test is assumed to have a planar surface and to deform in the plane of that surface. The television camera is mounted with its optical axis perpendicular to the surface of the test specimen. The camera must be connected to a computer containing a frame grabber, and the computer must be running image capture software. The experimental configuration is shown in Figure 13.

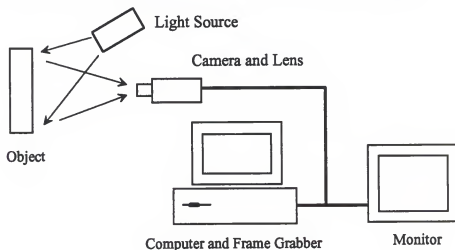


Figure 13. Experimental configuration.

Optics. Except where noted, all images were captured using a Fujinon Television zoom lens H6X12.5R, which is an $f/1.2$ lens with focal lengths ranging from 12.5 mm to 75 mm. The lens was mounted to the XC 77CE camera with a 10 mm extender ring, and was typically set at a focal length of 75 mm and focused at approximately 3 m. The actual object plane distances were smaller than 3 m due to the presence of the extender ring. This configuration placed the image plane approximately 48 cm from the object plane.

Experimental Procedure. The system shown in Figure 13 was assembled on a moveable base. Given a test specimen and a loading fixture, the assembly of the system

for a given test consists of aligning and focusing the camera. See the Appendix "The SPECKLE User Interface" for descriptions of the programs DT2953 and SPECKLE. The procedure for all experiments (except the aspect ratio calibration) described in this document is as follows:

1. If the test specimen has a dark or metallic surface then paint it with a light color spray paint.
2. Assemble the test specimen and its loading mechanism.
3. Position and focus the camera to obtain the desired image, aligning the optical axis of the camera lens with the normal to the surface of the test specimen.
4. Draw a dot pattern on the object using a felt tip pen.
5. Set the desired options for DT2953, as described in the Appendix, "The SPECKLE User Interface."
6. Use the dynamic histogram feature of DT2953 to determine the correct light source intensity and position, and the correct lens aperture setting.
7. For the first test with a given alignment of the camera, capture two images of the test specimen in its undeformed configuration and analyze the image pair using SPECKLE to determine the base line displacement and strain errors. If the results are not acceptable, then realign the camera and repeat this step until the results are acceptable.

8. Capture and save an image of the object in its undeformed configuration, load the object, and capture and save an image of the object in its deformed configuration.
9. Analyze the undeformed and deformed image pair to determine the desired displacements and strains.
10. To perform multiple repetitions of the current test, go to step 8.

Aspect Ratio Correction Calibration

SPECKLE's aspect ratio compensation mechanism was calibrated using three images of objects of known dimensions, which were imaged using different optical systems. The calibration results suggest the aspect ratio is dependent primarily on the combination of the CCD camera and the frame grabber [Sirkis, 1991]. The tests described below were all performed using the following procedure:

1. Image the test specimen with the lens under test.
2. Capture the image and save an image file using DT2953.
3. Load the image into SPECKLE and use the Scan Line Dialog Box to measure the horizontal and vertical dimensions, in pixels, of a feature of the object with known physical dimensions.
4. Use the resulting dimensions to compute an Aspect Ratio as described in the section "Aspect Ratio Calibration" of the Appendix.

Aspect Ratio Test 1

The first aspect ratio test was performed by imaging a two inch by two inch object using a television lens with a focal length of 75 mm. The resulting image was saved in the

file ASPECT1.SUB. The data from ASPECT1.SUB are shown in Figure 14. Note that the physical dimensions are in inches and the logical dimensions are in pixels.

File	ASPECT1.SUB				
Trial	Horizontal		Vertical		
	Pixels	Physical	Pixels	Physical	
1	283	2	321	2	
2	284	2	319	2	
3	284	2	319	2	
Average	283.6667	2	319.6667	2	
Ratio	1.12691				

Figure 14. Results of the first aspect ratio test.

Aspect Ratio Test 2

The second aspect ratio test was performed using a fixed focus lens with a focal length of 12.5 mm imaging a two inch by two inch object. The data from the second aspect ratio test are shown in Figure 15, where the physical dimensions are in inches and the logical dimensions are in pixels.

File	ASPECT2.SUB				
Trial	Horizontal		Vertical		
	Pixels	Physical	Pixels	Physical	
1	207	2	235	2	
2	206	2	235	2	
3	206	2	234	2	
Average	206.3333	2	234.6667	2	
Ratio	1.137318				

Figure 15. Results of the second aspect ratio test.

Aspect Ratio Test 3

The third aspect ratio test was performed using a zoom lens set at a 75 mm focal length with a 20 mm extender ring. The target object was 0.5 in by 0.5 in. The data are

shown in Figure 16, where the physical dimensions are in inches and the logical dimensions are in pixels.

File	ASPECT3.SUB			
Trial	Horizontal		Vertical	
	Pixels	Physical	Pixels	Physical
1	275	0.5	315	0.5
2	276	0.5	315	0.5
3	276	0.5	315	0.5
Average	275.6667	0.5	315	0.5
Ratio	1.142684			
Average Ratio	1.135637			

Figure 16. Results of the third aspect ratio test.

Summary Of Aspect Ratio Calibration

The average aspect ratio from the three aspect ratio tests is 1.135637. This value was entered into SPECKLE.INI as 1.13 to calibrate the Aspect Ratio Compensation Mechanism.

A Lucite Beam

The Lucite beam described in this section was used in both the rigid body translation and cantilever beam tests. This section describes the machining of the beam and the mechanisms used to translate it and to load it as a cantilever beam.

Machining The Beam

The Lucite beam was machined from a 0.36 inch thick sheet of Lucite. The sheet was rough cut on a band saw and then milled according to the specifications indicated in Figure 17. Note that the specified depth of the beam was 17/16 inch, but the final physical depth was 1.08 inch.

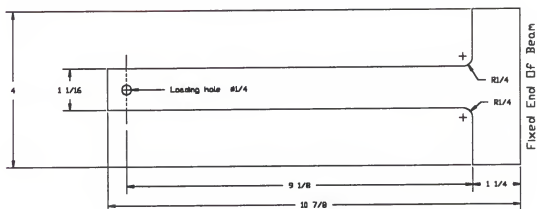


Figure 17. Mechanical specifications for the Lucite beam.

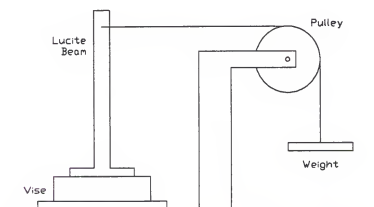


Figure 18. Loading mechanism for the cantilever beam.

Loading The Beam

The loading mechanism used to load the Lucite beam as a cantilever beam is shown schematically in Figure 18. A steel vise and a loading frame were bolted to an optical table and the Lucite beam was clamped using the vise. The loading frame was used to suspend a pulley with its top level with the loading hole of the beam. A 1/4 inch diameter bolt was placed through the hole in the beam and securely fastened to the beam

with washers and nuts. A multi-fiber steel cable was attached to hooks and installed as a loading cable with one hook attached to each side of the bolt, as close to the beam as possible. The loading cable was looped through a load holder, which was suspended from the pulley.

Translating The Beam

The Lucite beam was used as the test specimen in the rigid body translation test. The beam was clamped to a loading frame as shown schematically in Figure 19. The beam is clamped to the moving portion of the frame. The beam may be translated by turning the wheel on the left side of the schematic. The wheel is attached to a rod threaded at 18 threads per inch, which moves relative to the frame as it is turned and which is attached to the moving portion of the frame with a thrust bearing.

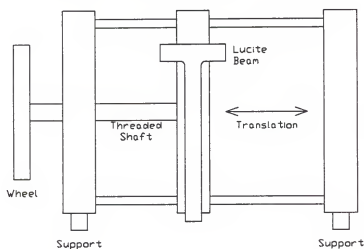


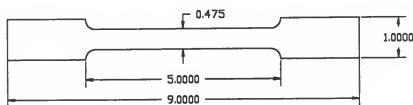
Figure 19. Rigid translation mechanism.

A Tensile Specimen

An aluminum tensile specimen was obtained and prepared for use in strain tests with SPECKLE. This section describes the mechanical specifications of the tensile specimen and the arrangement for loading it.

Mechanical Specifications

The tensile specimen is 0.040 inch thick and has physical dimensions as shown in Figure 20.



Aluminum Tensile Specimen.
All dimensions in inches.

Figure 20. Physical dimensions of the tensile specimen.

Loading The Tensile Specimen

The loading mechanism for the tensile specimen is shown schematically in Figure 21. The tensile specimen was placed in the wedge grips, and the loading weights were suspended from the right end of the lever arm. The mechanical advantage of the system was ten to one.

Shear Specimen

A shear strain demonstration device was used to evaluate the ability of SPECKLE to identify a shear strain field. The shear strain demonstrator was manufactured by

Ann Arbor Instrument Works and is self-loading. It has dimensions as shown in Figure 22. The shear strain demonstration device shears a 0.5 inch thick natural rubber sheet when the nut at the top of the frame is turned. The nut and the threaded shaft, which is attached to the member fixed to the center of the rubber sheet, are threaded at 28 threads per inch. Mechanical support for the shear strain demonstration device was provided by the loading frame used for the rigid body translation test.

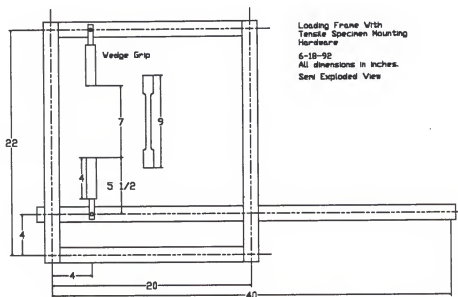


Figure 21. Loading the tensile specimen.

Summary

This chapter describes the preparation required for the experiments described in the next chapter. The aspect ratio calibration procedure need only be performed once for each camera and frame grabber combination. The remainder of this chapter describes the specimens and translation and loading systems used to produce known deformations for

tests designed to verify the accuracy of the PLS and PC displacement refinement methods, and SPECKLE's implementation of these methods.

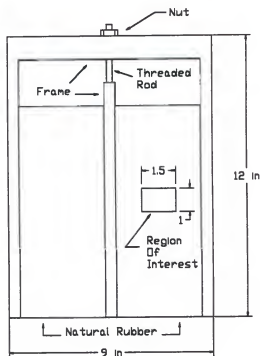


Figure 22. The shear demonstration device.

EXPERIMENTAL RESULTS

This chapter describes tests involving rigid and deformable body mechanics that were performed to provide verification of the PLS and PC methods of displacement refinement. Note that displacements reported by SPECKLE are given as ordered pairs of the form (right,down) with the components measured in pixels. The displacements are reported without correction for the aspect ratio of the video system. The right-down coordinate system is natural for use with computer graphics, but is not acceptable for reporting strain tensor components, which should be reported in a right hand coordinate system. In a personal communication, Dr. M.A. Sutton suggested the use of a down-right coordinate system as opposed to a right-up coordinate system. SPECKLE uses the down-right coordinate system for reporting strain tensor components. The horizontal components of both image coordinates and displacements are corrected for the aspect ratio of the video system before the strains are computed.

Rigid Body Translation

The first test of SPECKLE to be presented is a test involving a rigid body translation. The Lucite beam was painted with multiple coats of Krylon Flat White Interior and Exterior Enamel with sanding between coats, and dots were drawn on its

surface. Two of the three rows of dots are shown in Figure 23.



Figure 23. Dots on the Lucite beam for the translation test.

The test specimen was mounted to the moving portion of a loading frame that was in turn mounted to an optical table. The arrangement is shown in Figure 19. The loading frame is actuated by turning a ring mounted to a rod threaded at 18 threads per inch. The loading frame was mounted to the optical table so that the displacement produced by the frame was approximately horizontal. A dial indicator with a scale calibrated in 0.001 inch increments was mounted to measure the displacement of the object.

Experimental Procedure

The rigid body displacement tests were performed using the following procedure:

1. Move the test specimen to its reference position at which the dial indicator scale has been zeroed and capture the undeformed image.
2. Move the test specimen until the dial indicator reads 0.010 inches and capture the "deformed" image.

It is important when returning the test specimen to its reference position, to move it past the zero reading and then bring it back to the zero reading, since otherwise the slack in the

threads of the loading frame will cause inconsistent results when the test specimen is displaced to its "deformed" position.

Results

Three repetitions were performed, and the images were saved in files with root names RIGIDC, RIGIDD, and RIGIDE. The results from the three tests are shown in Figure 24 through Figure 26, respectively.

RIGIDC		Horizontal	Vertical
PC	Point 1	3.16E+00	-7.50E-02
	Point 2	3.19E+00	-3.22E-02
	Point 3	3.15E+00	-1.09E-01
PLS	Point 1	3.19E+00	-7.42E-02
	Point 2	3.21E+00	-2.20E-02
	Point 3	3.15E+00	-1.09E-01

Figure 24. RIGIDC displacement results.

RIGIDD		Horizontal	Vertical
PC	Point 1	3.09	-9.47E-02
	Point 2	3.05	-7.60E-02
	Point 3	3.05	-1.03E-01
PLS	Point 1	3.11	-9.73E-02
	Point 2	3.08	-5.72E-02
	Point 3	3.07	-9.24E-02

Figure 25. RIGIDD displacement results.

RIGIDE		Horizontal	Vertical
PC	Point 1	3.18	-1.17E-01
	Point 2	3.15	-7.61E-02
	Point 3	3.14	-1.33E-01
PLS	Point 1	3.16	-1.15E-01
	Point 2	3.10	-1.20E-01
	Point 3	3.11	-1.43E-01

Figure 26. RIGIDE displacement results.

Note that the displacements in the above tables are expressed in terms of pixels. The average of all horizontal components of displacements is 3.13 pixels. The width of the

beam is 1.08 inches, and the width of the image of the beam is 332 pixels. The physical displacement corresponding to the average horizontal displacement is then 0.01 inches. The error relative to the 0.010 inch displacement measured with a dial indicator is 1.81%. The results are graphed in Figure 27. The legend of Figure 27 shows the icons and line styles used to identify each line. The lines, except the average value line, are coded by the last letter of the test name and the method of computation of displacement. The average value line corresponds to the average of all indicated values. Note that a different line style is used for each test, and that most of the scatter appears to be between tests, rather than between the individual values for each given test.

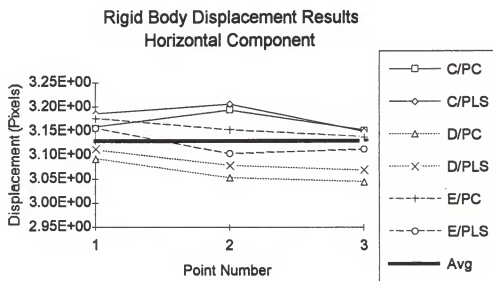


Figure 27. Rigid body displacement results.

Axial Strains In A Cantilever Beam

The second test of SPECKLE to be discussed is a measurement of the average strain due to bending in a cantilever beam. This test demonstrates the use of SPECKLE in situations involving a strain gradient in a known direction.

Experimental Arrangement And Procedure

The dot pattern shown in Figure 28 was applied to the beam with an overhead projector marker.



Figure 28. Dots for the cantilever beam test.

The beam was prepared for loading as shown in Figure 18. The camera was positioned normal to the face of the beam, and the camera lens was focused on the dot pattern. The undeformed images were captured with no load on the beam, and the deformed images were captured as soon as possible after the application of a ten pound load. The beam creeps under load, making the use of a pre-load impossible when using the DT-2953 frame grabber. The DT-2953 is not capable of holding multiple frames in its frame store memory, and the beam would creep significantly in the time it would take to save the

undeformed image to disk or machine memory in preparation for the capture of the deformed image.

Results

Three repetitions were performed. In each case an image was captured with the beam in its undeformed configuration, the beam was loaded and an image of the beam in its deformed configuration was captured. The load was removed from the beam as quickly as possible to minimize the effect of creep. The images corresponding to the three repetitions were saved in files with root names GRADA, GRADB, and GRADC, respectively. SPECKLE was used to compute the average strain along lines parallel to the center line of the beam on the tension side, along the neutral axis, and on the compression side. Both the PLS and PC methods were used to compute displacements that were subsequently used to compute the average strains. The resulting normal strains are shown in micro strain in Figure 29.

Test	Method	T	N	C
GRADA	PC	1332	-94	-1343
	PLS	1316	-120	-1376
GRADB	PC	1460	-109	-1481
	PLS	1482	-89	-1459
GRADC	PC	1379	-70	-1636
	PLS	1472	-65	-1515
Average	PC	1390	-91	-1487
	PLS	1423	-91	-1450
	Combined	1407	-91	-1468

Figure 29. Cantilever beam average normal strains.

Comparison With Beam Theory Predictions

A dial indicator was used to measure the elastic free-end displacement, d , of the beam in response to a 10 pound load, which is approximately 0.15 inch. Consider the

definition sketch shown in Figure 30.

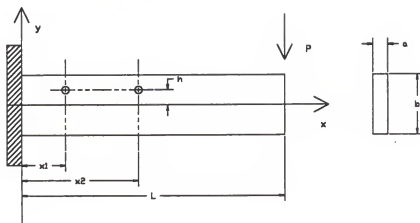


Figure 30. Definition sketch for the cantilever beam.

The two circles in the sketch represent the end points of a line along which the average strain is to be computed. It can be shown that the displacement of the free-end of a cantilever beam made of a homogenous elastic material with constant cross section is given by

$$d = \frac{PL^3}{3EI}$$

where P is the applied load, L is the length of the beam, E is the Young's Modulus of the material, and I is the centroidal moment of inertia of the area of the cross section of the beam. The above may be used to determine the elastic modulus in terms of the free-end displacement. It can further be shown that the axial strain due to bending is given by

$$\varepsilon(x, y) = -\frac{P}{EI}y(x - L)$$

The average strain along the line between the points (x_1, h) and (x_2, h) in the reference configuration is equal to the strain at the midpoint, which can be written as

$$\bar{\varepsilon}(x_1, x_2, h) = -\frac{P}{EI}h \left[\frac{x_2 + x_1}{2} - L \right]$$

The parameters for the line selected along the tension side of the beam are summarized in Figure 31. See Figure 30 for a definition sketch of the beam. The dot pattern used in the cantilever beam tests is shown in Figure 28. The values of x_1 and x_2 (see Figure 30) are the distances from the fixed end of the beam to the dots in the bottom and top, respectively, of the beam.

Beam Parameters	
a	0.36 in
b	1.08 in
x_1	2.25 in
x_2	3.25 in
L	9.25 in
h	0.40625 in
P	10 lb
d	0.15 in

Figure 31. Beam parameters.

Given the measured free-end displacement, the elastic modulus is 465 ksi and the corresponding theoretical value for the average strain on the tension side is then 1501 micro strain, which agrees well with the experimental results. A zero average strain is expected along the neutral axis, and along the compression side the average strain is expected to be the negative of the value computed along the tension side. The results are tabulated in Figure 29 and plotted in Figure 32. At each position on the beam, the columns from left to right in Figure 32 correspond to the legend entries from top to bottom. The legend entries are coded by the last letter of the test name and the method of displacement computation.

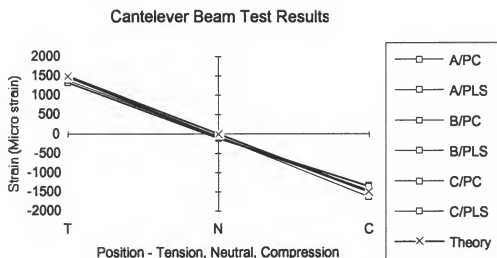


Figure 32. Axial strains for the cantilever beam.

Tension Test

This section describes a test in which the aluminum tensile specimen was loaded in pure tension, with the horizontal and vertical axes of the camera aligned with the principal axes of the test specimen. Recall that SPECKLE reports the strain tensor components in a down-right coordinate system. The dot pattern imaged for the principal axes tension test is shown in Figure 33. The pattern was imaged using the 75 mm zoom lens with a 20 mm extender ring. The zoom lens was set for a focal length of 75 mm and its focus was set at approximately one meter. The distance between the object and image planes was approximately nine inches.

Experimental Arrangement And Procedure

The tensile specimen was mounted as shown in Figure 21. The surfaces of the tensile specimen were painted with multiple coats of Krylon Flat White Interior and

Exterior Enamel with sanding between coats. The dot pattern is as shown in Figure 33.

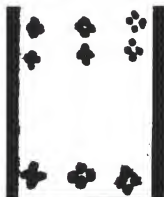


Figure 33. Dots for the tensile test.

The dots were drawn with an Overhead Projector Pen with an Extra Fine point. Each image pair was digitized as follows:

1. An 8 pound pre-load was placed on the loading arm to remove the slack from the loading frame, and the "undeformed" image was captured.
2. Two ten pound weights were placed on the loading arm and the deformed image was captured.

The mechanical advantage of the loading frame is 10:1, so the loads on the specimen were 80 lb and 280 lb, respectively. The calculations below are based on the principle of superposition and assume that the loads were zero and 200 pounds, respectively.

Results

Three repetitions of the procedure described above were performed. The images captured were saved in files with root names VERTE, VERTF, and VERTG. The data generated by SPECKLE are shown in Figure 34. The next section compares the measured

values with the theoretical values.

Test	Label	PC	PLS
VERTE	Edd	1198	1167
	Err	-434	-377
	Edr	-17	-55
VERTF	Edd	1145	1138
	Err	-300	-307
	Edr	-134	-101
VERTG	Edd	1156	1138
	Err	-371	-335
	Edr	-13	-11
Average	Edd	1166	1148
	Err	-368	-340
	Edr	-55	-56

Figure 34. Tensile specimen strain results.

Comparison With Theoretical Predictions

The physical dimensions of the aluminum tensile specimen are shown in Figure 20. The tension test parameters are shown in Figure 35, where E is Young's Modulus, P is the difference in the applied load between the final and reference configurations, t is the thickness of the specimen, and w is the width of the specimen.

Tensile Specimen Parameters	
E	1.00E+07 psi
P	200 lb
t	0.04 in
w	0.475 in

Figure 35. Tensile specimen parameters.

The nominal value for the incremental axial strain is given by

$$\epsilon_{dd} = \frac{P}{twE}$$

and Hooke's Law gives

$$\epsilon_{rr} = -\nu\epsilon_{dd}$$

where ν is Poisson's Ratio, which is 0.33 for Aluminum. The theoretical value for the

incremental normal strains are shown in Figure 36.

Theoretical Normal Strains	
Edd	1053 micro strain
Err	-351 micro strain

Figure 36. Theoretical strains.

The theoretical value for the shear strain is zero. The results are tabulated in Figure 34 and plotted in Figure 37. For each strain component in Figure 37, the columns from left to right correspond to the legend entries from top to bottom. The legend entries are coded by the last letter of the test name and the method of displacement computation.

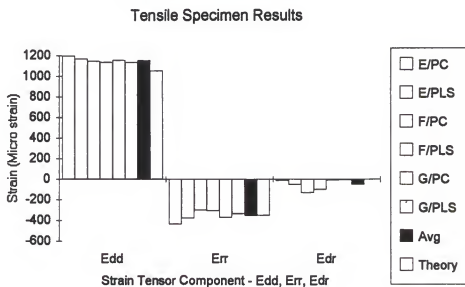


Figure 37. Tensile specimen strain results.

Shear Strain Test

This section describes a test performed using the shear strain demonstration device shown in Figure 22.

Experimental Arrangement And Procedure

The shear strain demonstrator was clamped to the loading frame used in the rigid body displacement test to provide mechanical support, and the dot pattern shown in Figure 38 was drawn on its surface. See Figure 38 for the location of the dot pattern on the shear strain demonstrator.



Figure 38. Dots for the shear strain test.

The camera was placed normal to the surface of the demonstrator and focused on the dot pattern. Each undeformed and deformed image pair was captured using the following procedure:

1. The wing nut on the shear device was loosened to completely unload the shear specimen. The wing nut was then tightened to remove slack and the undeformed image was captured.
2. The wing nut was tightened an additional one half turn and the deformed image was captured.

Results

Three repetitions of the procedure described above were performed. The resulting images were saved in files with root names DEMOB, DEMOC, and DEMOD. The strain tensor components as determined by SPECKLE are shown in Figure 39.

Test	Label	PC	PLS
DEMOB	Edd	170	165
	Err	248	261
	Edr	2419	2417
DEMOC	Edd	374	240
	Err	132	150
	Edr	2246	2263
DEMOD	Edd	238	190
	Err	474	504
	Edr	2127	2116
Average	Edd	260	198
	Err	285	305
	Edr	2264	2266

Figure 39. Shear strain test results.

Comparison With Theoretical Values

Consider the definition sketch shown in Figure 40, and the parameters shown in Figure 41. The shear strain may be estimated by considering the reduction in the right angle between material lines originally along the d and r axes. The tensor shear strain satisfies the equation

$$2\varepsilon_{dr} = \frac{u}{L}$$

where u is the shearing displacement and L is the width of the shear specimen. Given the parameters above, the expected tensor shear strain is 2232 micro strain, which agrees well with the experimental results. For each strain component, the results shown in Figure 42 correspond to the legend entries from top to bottom. The legend entries are coded by the

last letter of the test name and the method of displacement computation.

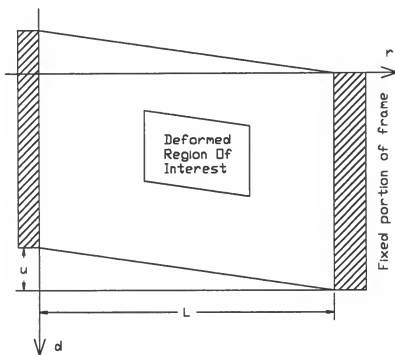


Figure 40. Definition sketch for the shear strain test.

Parameter	Value
Thread Pitch	28 thread/inch
Turns	0.5 turns
Displacement	0.018 inch
Width	4 inch

Figure 41. Shear specimen parameters.

Effects Of Out-Of-Plane Displacement

This section provides verification of the order of magnitude analysis of the effects of out-of-plane displacement on normal strains measured using vision systems when plane motions are assumed. The Lucite beam was mounted on a translation stage so that it could be moved parallel to the axis of the camera, which was fitted with the Fujinon zoom lens with a 10 mm extender ring and set for a 75 mm focal length, with the focus adjusted

at approximately 10 m.

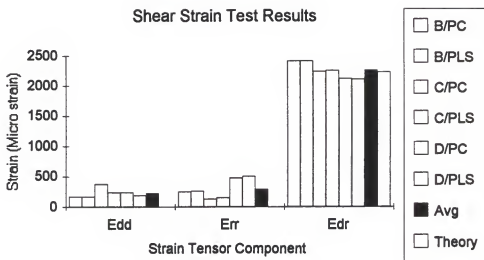


Figure 42. Shear strain test results.

The object to image plane distance was approximately 50 cm. The horizontal size of the field of view in the object plane was approximately 5.5 cm and the horizontal size of the sensitive area of the Sony XC-77 CE is 0.88 cm. The distance from the object to the pinhole equivalent to the lens, as described in the chapter "Analytic Results On Errors," is then 16.97 in. Two images of the object were captured with the object in its reference position. The object was displaced toward and away from the camera by 0.020 in, 0.030 in, and 0.040 in, and an image was captured in each position. The images were analyzed in pairs using the first reference image as the undeformed image in all cases. The resulting normal and shear strains are shown in Figure 43 that both normal strain components should agree with the theoretical normal strain, and the shear strain should be zero. The results confirm that the effects of out-of-plane displacement on normal strains are of the expected order of magnitude and that the effect of out-of-plane displacement on shear

strain is small, as expected. The equivalent pinhole model may prove to be useful for the estimation of allowable out-of-plane displacements.

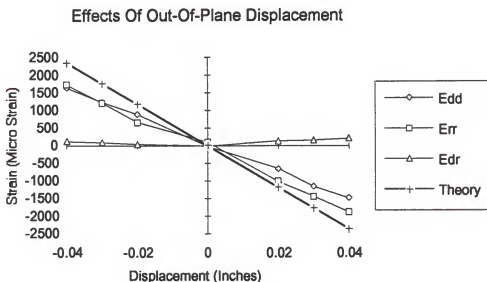


Figure 43. Effect of out-of-plane displacement on measured strains.

Summary

This chapter describes experiments which provide verification of the PLS and PC methods of displacement refinement, and of the use of these methods to estimate strains from displacements. Tests involving rigid body displacement, bending of a beam, and tensile and shear fields are described. Experience with using SPECKLE for the measurement of displacements and strains has indicated that it is imperative to analyze a zero motion test - called a rating test - to determine base line errors in displacements before proceeding to the desired measurements. It is possible to measure strains with a base line error of approximately one hundred micro strain, and rating tests provide a good means to estimate the base line strain errors.

CONCLUSIONS AND RECOMMENDATIONS

The primary objective of the current study was to develop an iterative technique for the determination of in-plane displacements and strains using digital images of a test specimen before and after deformation. This objective has been accomplished with the development of the Perturbation/Least-Squares (PLS) and Perturbation/Correlation (PC) methods for the iterative refinement of displacement, and the development of the program SPECKLE, which is a Microsoft Windows program that uses the PLS and PC methods to compute displacements and to estimate strains. This chapter lists conclusions that can be drawn from the results of the current study and recommendations for future work.

Conclusions

The scope of the current study includes the following:

1. The PLS and PC methods for the refinement of rigid body displacement estimates were developed as alternatives to the Newton-Raphson approach developed by Bruck.
2. Tests were performed to verify the PLS and PC method governing equations and the iterative displacement computation algorithm.
3. Qualitative analytic studies of the errors inherent in the use of vision systems for the computation of displacement and strain were conducted.

The remainder of this section presents some conclusions which can be drawn from the results of the current study.

Experimental Results

The experiments described in this document indicate that the PLS and PC methods can be used to determine in-plane displacements using digital images of a test specimen before and after deformation. The experiments also indicate that the resulting displacements can be used for the subsequent determination of normal and shear strains in a constant strain field, and average normal strains in a varying strain field. Typical errors in strain are on the order of 100 micro strain.

Analytic Studies

A secondary goal of the current study was to study analytic models of the effects of image noise on the use of correlation to measure displacements and strains using images of the test specimen before and after deformation. The results are summarized in the chapter "Analytic Results On Errors." The analytic studies of errors are consistent with published experimental and numerical studies, which conducted by other investigators. Correlation methods for the computation of displacements and displacement gradients function by locating extreme values of a correlation function [Sutton et al., 1988]. The analytic studies indicate that effects of noise on the displacements computed by correlation may be minimized by sampling the images of the test specimen at spatial frequencies that are high relative to the spatial spectrum of the images. The strain-displacement uncertainty relation derived for correlation predicts uncertainties in strain of the order of magnitude observed in studies in which correlation was used to determine strains and

displacements simultaneously. When the magnitude of the displacement gradient components are on the order of 0.001 and smaller, the results favor the computation of displacements neglecting the localized effects of displacement gradients and the subsequent determination of displacement gradients from the displacements computed in widely separated regions. The required distance between the centers of the individual correlation regions can be determined given the displacement error bounds and allowable strain errors. The difficulty with this approach is that it severely limits the allowable rigid body rotation between the undeformed and deformed configurations. The current solution to the problems of displacement gradient measurement as implemented by Sutton et al. [1991] involves the use of a finite element analysis to smooth the computed displacement fields. It would be preferable to be able to determine the displacement gradients from the acquired images without the computational overhead of finite element analysis and the required assumptions about the constitutive behavior of the test specimen material.

Recommendations For Future Work

This section contains recommendations based on the author's experience and results published by other researchers.

Verification Of The PLS And PC Methods

The results of the experiments described in this document indicate that the PLS and PC methods of displacement refinement can be used to measure in-plane displacement and strain. The next logical step is to explore further how the PLS and PC methods compare with each other, with the Newton-Raphson approach to Digital Correlation developed by Bruck et al. [1989], and with a coarse-fine approach to Digital Correlation

as developed by Peters et al. [1983] and Chu et al. [1985]. The comparisons should consider accuracy, computational efficiency, experimental convenience and hardware requirements.

Extensions Of The PLS And PC Methods

The PLS and PC methods for displacement refinement should be extended to include rigid body rotation. One way to support rigid body rotation is to include displacement gradients in the approximate motion used to derive the governing equations for the methods. The results published by Sutton et al. [1991], the results of the current study, and the author's experience indicate that the extension to computing displacement gradients will be more difficult than simply including displacement gradients in the equations. The PLS method was originally developed with strains and rigid rotation included in the least-squares fit. Numerical tests of the original PLS method governing equations demonstrated that the measured strains and rotation were unacceptably sensitive to the computed rigid body displacement. The current versions of the PLS and PC methods are limited to cases involving rotations of at most a few degrees. The largest rigid rotation that can be tolerated by the current versions is not known and should be determined. This problem has been considered for Displacement-only Correlation based on a coarse-fine search algorithm [Chen and Chiang, 1992].

Rigid Rotation Tolerance

The PLS and PC methods of displacement refinement have been implemented neglecting the local effects of displacement gradients over the regions in which center point displacements are to be computed. Strains are computed from the center point

displacements in regions sufficiently far apart that the expected error in displacement divided by the distance between the region center points is small compared to the strain to be measured. The following factors contributed to the decision to neglect the localized effects of displacement gradients:

1. The author's negative experience with an early version of the PLS method which included displacement gradients in the least squares fit.
2. The conclusion by Sutton et al. [1991] that displacement smoothing is required for accurate determination of displacement gradient components.
3. Estimates based on the strain-displacement uncertainty relation derived in this document that indicate a total loss of precision on displacement gradient calculations when scales typical in the current study are used.
4. Estimates based on the strain-displacement uncertainty relation that indicate the errors in displacement due to the neglecting of displacement gradients should be on the order of 0.01 pixels.

The most severe limitation imposed by neglecting displacement gradients is that rigid rotations between exposures must be kept very small.

Out-Of-Plane Displacement Detection

Analytic and experimental results presented in this document indicate that the normal strains measured using SPECKLE are affected by out-of-plane displacement of the test specimen. The errors in normal strains are on the order of the out-of-plane displacement divided by the distance between the object and image planes. It should be

possible to detect such out-of-plane displacements by attaching a long thin rigid object to the test specimen. The rigid object should have a dot pattern drawn on it, and be attached to the test specimen at only one point. The rigid object will move with the test specimen, but not deform with it. The average strain along the axis of the rigid object should be computed. If it is significant, then out-of-plane motion should be suspected.

Modification Of SPECKLE

SPECKLE is a Microsoft Windows 3.x application that implements the PLS and PC methods of displacement refinement. This section lists some enhancements to the program which should be implemented before more data is processed.

Efficiency and usability issues. The current version of SPECKLE requires that the user mark exactly three points, after which the following computations are performed:

1. An average displacement is computed in a region centered about each marked point.
2. Normal strains are computed along three lines defined by the selected points.
3. Strain transformation equations are used to determine normal and shear strains in a down-right coordinate system.

This structure can be inefficient. The cantilever beam test described in the chapter "Experimental Results" did not require the computation of the general state of strain, but these calculations were always performed. The current implementation should be extended as follows:

1. The user should be able to specify the number of points to be marked.
2. The user should be able to associate pairs of points marked for displacement calculations for calculation of normal strains.
3. The user should be able to specify names for the individual normal strains.
4. The user should be able to associate triads of normal strains for the computation of strain fields.
5. The user should be able to control the format of the log files SPECKLE writes to facilitate compilation of experimental results.

SPECKLE is an interactive program that allows the user to control the points at which displacements are computed and the parameters used to control the iteration. The current version of SPECKLE sometimes wastes time by computing quantities that should have been saved for reuse. The primary advantage of building collections of computed results is that only the results affected by changes made by the user would need to be computed when the data base is updated to reflect changes made by the user. Such modifications would allow the collection and processing of much more data on the performance of the PLS and PC methods than has been included in this study.

Extended support for research. SPECKLE currently supports only bilinear interpolation and should be extended to support non-linear interpolation methods. SPECKLE should also be extended to provide options for the method used to estimate derivatives of the deformed image. The use of arbitrary collections of displacement values described above would allow SPECKLE to support the calculation of strains and small

rigid rotations by the technique used by Sirkis and Lim [1991]. Sirkis and Lim computed strains by analytic differentiation of surfaces fit to displacements computed at points arranged in a rectangular grid. The experimental work performed by Sutton et al. [1991] and the strain-displacement uncertainty relationship derived in the current study indicate that strains obtained directly from digital correlation are not sufficiently accurate to allow measurement of strains smaller than one percent. The strain calculation method used by Sirkis and Lim should be provided as an option to users of SPECKLE.

Summary

The results of the current study include the development of alternate iterative techniques for the computation of in-plane displacements using digital images of a test specimen before and after deformation. The resulting displacements may then be used to compute estimates of normal and shear strains in a constant strain field or average normal strains in a varying strain field. The program SPECKLE, developed by the author as part of the current study, supports these computations with a graphical user interface under the Microsoft Windows environment. The analytic results on errors in the use of vision systems are consistent with numerical and experimental studies published by other investigators. Experiments indicate that the experimental system developed as part of the current study may be used to measure normal and shear strains to within approximately one hundred micro strain.

APPENDIX

THE SPECKLE USER INTERFACE

The purpose of this Appendix is to describe the user interface of SPECKLE, which is a Microsoft Windows 3.x application that supports the computation of in-plane displacements and strains using digitized images of a test specimen before and after deformation. Displacements may be computed by either of the PLS or PC methods, which are described in this document. Strains are computed as average normal strains along three user-specified lines. The average normal strains are transformed to normal and shear strains in coordinates natural to the orientation of the camera used to form the images. SPECKLE does not support compensation for lens distortions, but it does support compensation for the image stretching observed when the camera and frame grabber are not synchronized [Sirkis, 1991]. This appendix begins with a description of the frame grabber operation program, which provides a real-time histogramming feature to assist the user in the selection of the correct exposure. The user interface of SPECKLE is described in detail from a user's perspective. This appendix concludes with a description of the technique for calibrating SPECKLE's aspect ratio compensation mechanism.

The Microsoft Windows Environment

MS-DOS, Microsoft Windows and Microsoft Windows Software Development Kit (SDK) are Registered Trademarks of Microsoft Corporation. Microsoft Windows provides a graphical user interface for MS-DOS based personal computers. This section

provides a brief overview of the Microsoft Windows environment from the user's perspective. A detailed description of the environment is beyond the scope of this document. Petzold [1990] provides an excellent introduction to the Microsoft Windows environment from a programmer's perspective. Detailed information can be found in the SDK documentation [Microsoft, 1990a] and [1990b]. The overview of the Microsoft Windows environment uses the main window of DT2953, shown in Figure 44, as an example.

Windows

The Microsoft Windows user interface is an object oriented system [Petzold, 1990, p. 13]. Almost everything that appears on the video display is a window. Figure 44 shows an example of a window displayed by Microsoft Windows version 3.x.

Dialog Boxes

Dialog boxes [Petzold, 1990, p 403] are specialized windows that are often used to allow the user of a program to control the execution of the program. The window shown in Figure 44 is a dialog box. The purpose of this section is to introduce some terminology which is required to describe the use of the DT2953 operation program and of SPECKLE.

Push Buttons. The rectangular region marked Save File in Figure 44 is a push button. A left mouse button click on a push button generally causes the program to perform some task. To "push" a button, move the mouse cursor until it is over the button and quickly press and release the left mouse button.

Check Boxes. The Options group on the DT2953 main window contains buttons marked with the labels Floating Point and Multi Save. The square next to the Floating Point label is clear and the square next to Multi Save is checked. The Floating Point option is not selected and the Multi Save option is selected. Check Boxes are often used to allow the user to make independent choices from a collection of options which may be either on or off. The state of a Check Box may be toggled by a mouse click.

Radio Buttons. The Active File group on the DT2953 main window contains three Radio Buttons marked File 1, File 2, and File 3 respectively. In Figure 44, the File 1 Radio Button is selected and the other two are not selected. Radio Buttons are often used to allow the user to select exactly one of a collection of mutually exclusive options. A mouse click on a selected Radio Button will not change its state, and a mouse click on an unselected Radio Button will deselect the currently selected Radio Button and select the clicked Radio Button.

Edit Fields. The Options group on the DT2953 main window contains an edit field marked Average. To change the contents of an edit field, click on it with the mouse to activate it and then edit the text it contains.

The Frame Grabber Operation Program

The frame grabber operation program, DT2953, is a Microsoft Windows 3.x application which supports the Data Translation DT-2953 frame grabber for the IBM PS/2 Micro Channel Architecture. The DT-2953 is an eight-bit frame grabber capable of storing a single frame in its frame-store memory. This section describes the user interface

of DT2953. The main window of DT2953 is shown in Figure 44.

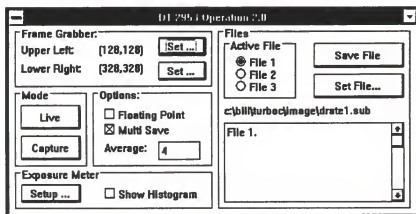


Figure 44. The DT2953 Main Window.

The controls on the DT2953 main window are as follows:

1. The Frame Grabber group controls allow the user to specify the portion of the data from the frame grabber to be saved in image files.
2. The buttons in the Mode group place the frame grabber in live video and capture modes.
3. The Options group controls the generation of output data.
4. The Exposure Meter group controls the real-time histogram feature.
5. The Files group allows the user to specify the names and comments for the output files, and to save image files.

The remainder of this section describes the DT2953 controls in detail.

The Frame Grabber Group

The Frame Grabber group box displays the coordinates of the upper left and lower right pixels to be saved as part of the output image files. The Set button beside each point displays a dialog box which allows the user to edit the corresponding point. The values set are written to the file DT2953.INI, so they persist between sessions.

The Mode Group

The Mode group box contains buttons marked Live and Capture, which place the frame grabber in Live Video and Capture modes, respectively. The frame grabber is placed in Live Video mode each time DT2953 is loaded.

The Options Group

The Options group box contains the Multi Save checkbox and the Average edit field. The Average field allows the user to specify an integer number of frames to be averaged together to form a single output image. Multiple frames are averaged to form a single image only if the Multi Save check box is selected. The details of the image save process are described below.

The Exposure Meter Group

The Exposure Meter group box contains the Show Histogram checkbox, which allows the user to display a popup window containing a histogram of a selected region of the frame grabber. The frame grabber is scanned in idle time [Petzold, 1990, p. 599] and the histogram window is updated each time a scan is completed. The histogram is updated at a rate of approximately two times per second. The histogram window is useful

for the selection of proper light levels for the output images. The user may adjust either the light source intensity or the imaging lens aperture and see the effect on the image histogram in real time. The dynamic histogram window, illustrated in Figure 45 shows a plot of the number of pixels at each gray level versus gray level.

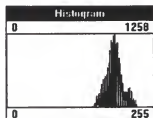


Figure 45. The dynamic histogram window.

The Files Group

The Files group box contains controls which allow the user to specify the file names and comments for the output image files, and to save the output images. The Active File group box contains radio buttons corresponding to three files. It is recommended to capture two images of the object in its undeformed configuration and one image of the object in its deformed configuration. The two images of the object in its undeformed configuration should be analyzed to determine base line errors. The remaining controls in the Files group box affect the file indicated by the selected radio button.

Setting the output file name. The name of the file to which output data is to be written is displayed directly below the Active File group box. The Set File button displays a dialog box which allows the user to specify the name of the output data file.

Editing the file comment. The file format used by DT2953 and SPECKLE allows the inclusion of a comment of up to approximately 500 characters. The comment is initialized to indicate the file number, and may be edited to contain any test-specific data desired by the user. The comments of both the undeformed and deformed images are included in the reports generated by SPECKLE.

Saving image files. The Save File button saves the contents of the DT-2953 frame store memory to the file specified for the active file. If the Options | Multi-Save checkbox is unchecked, then the frame store memory is scanned and written to the indicated file without averaging. If the Options | Multi-Save checkbox is checked, then multiple frames are captured and averaged to form the output image. The number of frames to be averaged is specified in the Options | Average edit field.

Overview Of The SPECKLE User Interface

This section provides an introduction to the SPECKLE user interface. SPECKLE conforms to the Multiple Document Interface (MDI) specification [Petzold, 1990, p. 853] with some minor exceptions, as noted below. The purpose of this section is to introduce the types of windows available in SPECKLE. The use of SPECKLE for the computation of displacements and strains is described in a subsequent section of this appendix.

Image Viewers

When SPECKLE is first loaded by Microsoft Windows it displays two MDI Child Windows titled "Undeformed Image" and "Deformed Image," respectively. The first step in the normal use of SPECKLE is to load the two source images to be analyzed. To load

images, select File | Load. SPECKLE displays two file load dialog boxes in succession. The first file selected corresponds to the undeformed configuration, and the second file corresponds to the deformed configuration. Pressing the cancel button on either dialog cancels the load operation and retains any currently loaded images. If two images are selected, a delay of approximately one minute occurs while SPECKLE builds bitmaps corresponding to the contents of both images. The user may move and resize the viewers, as well as use the scroll bars to scroll to any desired location in the images. The main difference between SPECKLE and normal MDI applications is that SPECKLE requires that exactly one Undeformed Image Viewer and exactly one Deformed Image Viewer exist at all times.

Histograms

To view the histogram for an image, select its viewer as the active window and select Launch | Histogram. SPECKLE creates a new MDI Child window containing the graph of the histogram. Histograms can be very useful in the identification of images that are too dark or that may be clipped due to overexposure.

Scan Lines And The Scan Line Dialog Box

The Scan Line feature of SPECKLE can be used to view a cross-section of the gray level distribution of an image along an arbitrary line. The Scan Line Dialog Box can also be used to determine the exact location (in pixels) of a given point as well as to measure the distance (in pixels) between two points.

Line selection. To select a line across an image, proceed as follows:

1. Select the viewer containing the image as the active window.
2. Choose the menu option Launch | Scan Line to display the Scan Line Dialog Box for the selected image.
3. Move the mouse cursor to the first point on the desired line and press and hold the left mouse button.
4. Drag the mouse cursor [Microsoft, 1991, p. 15] to the other end of the desired line and then release the left mouse button. At this point the coordinates of the two endpoints of the line chosen are displayed in the edit fields of the Scan Line Dialog Box. Note that it is also possible to edit the coordinates of the two end points by clicking on or tabbing to the corresponding edit fields and typing the desired values.

Scan line graph creation. To create a window containing the graph of an image scan line, proceed as follows:

1. Select the desired line as described above.
2. Press the Ok button on the Scan Line Dialog Box.

The Scan Line Dialog Box will disappear and a new MDI Child window containing the desired graph will appear in the foreground.

The Scan Line Dialog Box Controls. The Scan Line Dialog Box has edit controls which allow the user to modify the coordinates of the end points of the selected line, launch a Scan Line Graph MDI child window, and display a message box with the length

of the selected line.

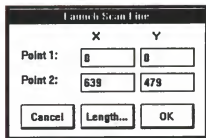


Figure 46. The Scan Line Dialog Box.

The Scan Line Dialog Box, shown in Figure 46, contains the following controls:

1. Edit fields for the coordinates of the endpoints of the selected line.
2. The Cancel button which closes the dialog box.
3. The Length button which displays a message box [Microsoft, 1990b, p. 4-306] containing the length of the selected line in pixels.
4. The Ok button which launches a Scan Line Graph MDI Child Window for the selected image and line.

Computation Of Displacements And Strains

The use of SPECKLE to compute the average state of strain in a region of the undeformed image involves the following steps:

1. Load the undeformed and deformed images.
2. Mark the points of interest in both the undeformed and deformed configurations to indicate the base displacements.
3. Initiate the computation of displacements and strains using the Strain Dialog Box.

The user has the option of logging results after they have been computed, as described below. The remaining parts of this section provide detailed descriptions of the process for determination of base displacement estimates and the use of the Strain Dialog Box for the computation and logging of results.

Determination Of Base Displacements

The SPECKLE user interface provides support for the marking of three pairs of points. For each pair, one point is in the undeformed image and other point is in the deformed image. The points in the deformed image provide an estimate of the motion in the neighborhood of the corresponding points in the undeformed image. The resulting displacement estimates are used as seed values for an iterative displacement refinement process. This section contains a detailed description of a general procedure for determining base displacement estimates. It should be noted that the steps indicated are intended as a guide only. In some cases the user may be able to easily identify features in both images and simply click on them to indicate their positions. In other cases, the user may find that the estimates provided to SPECKLE are not accurate enough for the iteration to converge, in which case additional steps involving the Transfer Dialog Box, described below, are required. The general procedure for the determination of base displacements is as follows:

1. Mark identifiable points in the undeformed image.
2. Transfer first estimates of the points corresponding to those marked in the undeformed image to the deformed image.

3. Correct the points marked in the deformed image.
4. Move the marks in the undeformed image to the actual points of interest.
5. Restore the previous displacement components to update the marks in the deformed image.

Schematic diagrams. The description of the use of SPECKLE is accompanied by schematic diagrams. The diagrams are all similar to the schematic diagram shown in Figure 47.

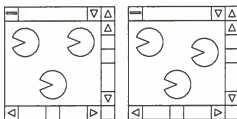


Figure 47. Schematic of the image viewers.

The schematic diagram shown in Figure 47 represents the undeformed and deformed image viewers with images loaded. Each image contains three symbols, each of which represents a white light speckle. The speckle symbols have the following components:

1. The outer circle represents the boundary of the speckle.
2. The pie-slice in the boundary represents an easily identified feature on or near the boundary.

Note that if the user is content to compute displacements based on the easily identified points then it is sufficient to mark them in both windows using the marking modes described below and proceed to the strain analysis. The problem is that often the points

which are easily identified are poor choices for displacement computation, due to small gray level differences in the neighborhood of the point. In the following it is assumed that the user wishes to compute displacements using points near to but distinct from those which can be easily identified.

Mark identifiable points on the undeformed image. For each point of interest in the undeformed image, locate a nearby feature that has a clearly identifiable counterpart in the deformed image. Mark the features by applying the following procedure to each of the three points:

1. Choose the menu option for the marking mode corresponding to the current point.
Note that the mouse cursor changes to indicate which point is to be marked.
2. Click on the clearly identifiable feature. A color-coded selection box appears around the point.

The schematic diagram appropriate to the current situation is shown in Figure 48.

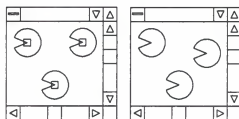


Figure 48. Marks on the undeformed image.

Transfer first estimates of corresponding points. It is now necessary to mark the features in the deformed image which correspond to those marked in the undeformed image. The recommended procedure is as follows:

1. Choose the menu option Modes | Transfer to display the Transfer Dialog Box.

The Transfer Dialog Box is a multi-purpose dialog which may be used to mark points in the deformed image based on the locations marked in the undeformed image. The Transfer Dialog is shown in Figure 50.

2. Choose one of the points to be the Active Point by selecting the appropriate radio button in the Active Point group box, and check the Set All check box. A mouse click on the deformed image sets the location in the deformed image of the Active Point, and the displacement estimate components are determined from the location in the two images. When Set All is checked the coordinate of the two points which are not active are also updated using the same base displacement as determined for the Active Point.

The schematic diagram of the current situation is shown in Figure 49. Note that in the schematic of Figure 49, one of the points in the deformed image is not marked correctly. It will be corrected in the next step.

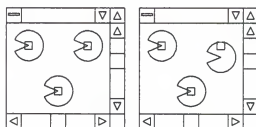


Figure 49. First displacement estimate.

Set Base Displacements		
Base Displacements		
	Horizontal	Vertical
Point 1 (RED)	0	0
Point 2 (GREEN)	0	0
Point 3 (BLUE)	0	0

Mouse Options	
Active Point ☒ Point 1 ☐ Point 2 ☐ Point 3	Set Options ☐ Set All

Figure 50. The Transfer Dialog Box.

Correct marks on deformed image. The flexibility of the Transfer Dialog Box allows options for completion of the current step. For each point which is not marked correctly in the deformed image perform one of the following steps using the Transfer Dialog Box:

1. Uncheck Set All, select the current point as the Active Point and click on the new location for the point in the deformed image.
2. Edit one or both of the components of the displacement for the current point and press the Set button to update the mark in the deformed image. Note that changes made to the contents of the edit fields are not reflected on the deformed image viewer until the Set button is pressed.

The schematic diagram appropriate for the current situation is shown in Figure 51.

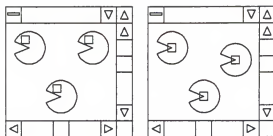


Figure 52. Updated undeformed image marks.

Restore previous base displacements. At this point it is assumed that the user is unable to identify the correct locations for the marks in the deformed image, since if the user had been able to do so then most of the above steps would have been skipped. The Transfer Dialog Box provides a displacement restoration feature which simplifies the process of updating the deformed image marks. Perform the following steps:

1. Choose the menu option Modes | Transfer to display the Transfer Dialog Box.
2. Press the Restore button. Note that the deformed image marks have moved to points which are displaced through the previously determined base displacement. The Transfer Dialog Box always saves the last set of base displacement vectors which were set either using the Set button or by mouse clicks on the deformed image viewer. The Restore feature places the deformed image marks at the current locations of the undeformed image marks plus the last set displacement.

The schematic diagram appropriate to the current situation is shown in Figure 53.

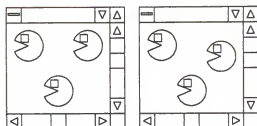


Figure 53. Marks after displacement restoration.

The marks in the deformed image are set to the user's best estimate of the locations corresponding to those marked in the undeformed image.

Comments on iteration failure. The next section describes the use of the Strain Dialog Box for the computation of displacements and strains based on the points marked as described in this section. The iterative displacement refinement process may fail at one or more of the points. In some cases inspection of the iteration results indicates that the initial estimate of displacement may have been in error. Should this situation arise, close the Strain Dialog Box and open the Transfer Dialog Box. Edit the appropriate displacement components and press the Set button. Then close the Transfer Dialog Box and repeat the strain computation. The power of the Transfer Dialog Box is most evident during the correction of problems with the convergence of the displacement correction iteration.

The Strain Dialog

The Strain Dialog Box provides the user with the controls necessary to initiate the computation of the state of strain, and to set parameters which control the computation of

displacements and strains. The Strain Dialog Box is shown in Figure 54.

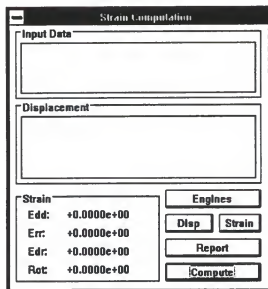


Figure 54. The Strain Dialog Box.

The controls on the Strain Dialog Box are as follows:

1. The Input Data and Displacement Iteration list boxes are used to display information relating to the current computations.
2. The Strain group box contains static controls which display the results of the current strain computation.
3. The Engines button displays the Engines Dialog Box which allows the user to choose the type of displacement and strain calculations to be performed.
4. The Displacement button displays a setup dialog box for the currently selected Displacement Engine.
5. The Strain button displays a setup dialog for the currently selected Strain Engine.

6. The Compute button initiates the computation of the state of strain.
7. The Report button displays a dialog box which allows the user to append a summary of the current results to a text file.

SPECKLE Configuration Control

This section describes the portion of the SPECKLE user interface which is dedicated to the control of calculations. The dialog boxes described in this section must be accessed from the Strain Dialog Box, which is described in the previous section.

Computational Engines Dialog

The Strain Dialog Box is the primary user interface element during the actual computation of displacements and strains. The computations are initiated when the user presses the Compute button on the Strain Dialog Box. SPECKLE computes strains by first computing displacements over regions of user-controllable size. The effects of strain over the small regions of displacement fit are neglected in the computation of the center point displacements. The strains are then determined from the center point displacements of regions sufficiently far apart that the strain may be calculated accurately. The design of SPECKLE allows the user to independently select the method to be used to compute displacements and strains. The available methods of computation are each represented by an object called a Computational Engine. Computational Engines are classified as Displacement Engines and Strain Engines. The active Displacement Engine computes displacements under direction of the active Strain Engine which subsequently uses the resulting displacements to estimate strains. The Computational Engines Dialog allows the user to specify the type of calculations to be performed when the Compute button is

pressed. The Engines Dialog Box is shown in Figure 55.

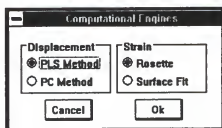


Figure 55. The Computational Engines Dialog Box.

Overview Of Engine Dialog Boxes

SPECKLE allows the user to select displacement and strain calculation methods through the selection of the corresponding Computational Engines. This arrangement simplifies the design of SPECKLE, and allows for future enhancements. The Engine Setup Dialog Boxes, displayed when the user presses the Disp or Strain buttons on the Strain Dialog Box, allow the user to set values of parameters which control SPECKLE's calculations.

Displacement Engine Dialogs

SPECKLE provides Displacement Engines for both the PLS and PC methods of displacement correction. The PC Engine Setup Dialog Box is shown in Figure 56.

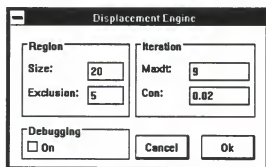


Figure 56. The Displacement Engine Dialog Box.

The controls on the PC Engine Dialog Box are as follows:

1. The Region | Size edit field allows the user to specify the base size of the region to be used in the determination of displacements.
2. The Region | Exclusion edit field allows the user to specify a number of pixels to be rejected near the border of the region to be used to determine displacements. The exclusion of data near the edges is required since the finite difference differentiation algorithm does not produce valid derivative values near the edge of the region.
3. The Iteration | MaxIt edit field allows the user to specify an upper limit for the number of iterations to be used to correct a displacement. If the iteration fails to converge within the specified number of iterations, the message "Max Iterations Exceeded" is placed in the iteration list on the Strain Dialog Box.
4. The Iteration | Con edit field allows the user to specify the magnitude of the difference between approximate displacements that is allowed at convergence. The value is specified in pixels.
5. The Debugging | On checkbox is used as a signal for SPECKLE's internal debugging mode. The Displacement Engines currently have the ability to generate test data for debugging purposes. This feature will most likely be removed in future versions of the program.
6. The Ok and Cancel buttons cause the values entered in the dialog to be accepted and ignored, respectively.

The PLS Engine Dialog Box contains all of the controls of the PC Engine Dialog Box, and provides additional controls to allow the user to control the discrimination of data. The PLS Engine Dialog box is shown in Figure 57.

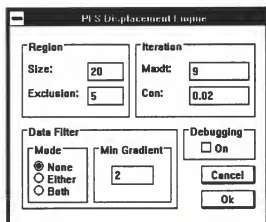


Figure 57. The PLS Engine Dialog Box.

Since the PLS method is based on a least squares fit directly on the values of the undeformed and deformed images it is natural to allow values which are known to be poor choices for use in the fit to be discarded from the fit. The order of magnitude analysis provided in the chapter "Analytic Results On Errors" is the basis for the PLS Displacement Engine Data Filter. The effects of noise on displacement determination increase as the derivative of the gray levels in the images decreases. The PLS Displacement Engine therefore provides the option to reject data from pixels at which the estimated partial derivatives are smaller in absolute value than a user-supplied constant. The data filter supports the following modes of operation:

1. Filter None, which accepts all data points.

2. Filter Either which accepts an individual datum if the absolute value of either estimated partial derivative of the deformed image at the point exceeds the threshold value.
3. Filter Both which accepts an individual datum only if the absolute values of both estimated partial derivatives of the deformed image at the point exceed the threshold value.

The controls in the Data Filter group box on the PLS Displacement Engine Dialog Box allow the user to control the settings of the PLS Engine Data Filter.

Strain Engine Dialogs

The Rosette Strain Engine is recommended for normal use due to current limitations in the protocol for communication between the active Strain Engine and the SPECKLE Image Viewers. The Rosette Engine currently does not define a setup dialog box. The current form of the user interface of SPECKLE provides only minimal support for the user of the Surface Fit Engine, since the only point which may be marked is the center of the region of interest. There is currently no support for the adjustment of the base displacement to be applied at individual points in the region of interest. The Rosette Engine Setup Dialog Box is shown in Figure 58.

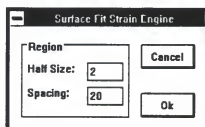


Figure 58. The Surface Fit Engine Dialog Box.

The Surface Fit Strain Engine computes strains by analytic differentiation of a surfaces fit to both components of displacement at each point in a square grid of points. The number of points and the spacing between them is user-controllable. The controls on the Surface Fit Strain Engine Dialog Box are as follows:

1. The Regions | Half-Size edit field allows the user to specify the number of points to be used in the surface fits. The actual number of points used is one plus twice the half size.
2. The Region | Spacing edit field allows the user to specify the spacing, in pixels, between the points on the grid.

The grid points are centered around the point marked as Point 1 on the undeformed image viewer.

Aspect Ratio Calibration

The file SPECKLE.INI is a text file which contains setup information for SPECKLE. The SPECKLE user interface provides dialog boxes allowing the user to set almost all parameters without the need to edit the initialization file. The only parameter which must be set directly in the initialization file is the value of the aspect ratio for the camera and frame grabber combination. This section describes procedures for the determination of the aspect ratio, and for the subsequent communication of the value obtained to SPECKLE.

Aspect Ratio Measurement

The aspect ratio for the camera and frame grabber combination may be calculated from manufacturer's specifications for the camera and frame grabber [Sirkis, 1991], or measured directly. This section is concerned with the direct measurement of the aspect ratio. The basic procedure is to capture an image which contains length references in both the horizontal and vertical directions. The length references should span as much of the frame width and height as possible. Suppose that the horizontal reference physically has width, w , and occupies W pixels horizontally. Suppose that the vertical reference physically has height h , and occupies H pixels vertically. The number of pixels spanned by each length reference can be determined by loading the reference image into SPECKLE and using the Scan Line Dialog as described earlier in this appendix. Assume that the observed lengths W and H are proportional to the physical lengths w and h and given by

$$\begin{aligned} W &= \frac{1}{\alpha} \beta w \\ H &= \beta h \end{aligned}$$

where α is called the aspect ratio and β is a constant related to the magnification of the optical system. Then the aspect ratio is given by

$$\alpha = \frac{H}{h} \frac{w}{W}$$

SPECKLE uses the aspect ratio to correct logical coordinates and displacements before computing strains.

Calibration Of SPECKLE

The aspect ratio should be substituted for the default value of 1.0 in the file SPECKLE.INI at the lines that read

```
[Strain]
xOverYRatio=1.0
```

The Microsoft Windows Programmer's Reference contains detailed descriptions of initialization files [Microsoft, 1990b, p. 4-199]. Note that displacements indicated by SPECKLE in the strain dialog are not corrected for the aspect ratio. All strains reported by SPECKLE are corrected for the aspect ratio.

Summary

This appendix presented an overview of the user interface of SPECKLE, which is an interactive program. The procedures described in this appendix should not be considered limiting. It should also be noted that the full procedure of marking points in both images, remarking the undeformed image and restoring the previous displacements is often unnecessary. An image which has good characteristics for use in the determination of displacements and strains has sharp features which are easily identifiable. The user's indication of the base displacement is often sufficiently accurate for the iteration to converge.

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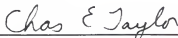
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BIOGRAPHICAL SKETCH

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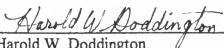
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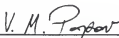


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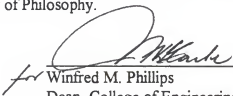

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December, 1992


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